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A real-time neural network model, called affective balance theory, is developed to explain many properties of decision making under risk that heretofore have been analyzed using formal algebraic models, notably prospect theory. The model describes cognitive-emotional interactions that are designed to ensure adaptive responses to environmental demands but whose emergent properties nonetheless can lead to paradoxical and even irrational decisions in risky environments. Emotional processing in the model is carried out by an opponent processing network called a *gated dipole*. Learning enables cognitive representations to generate affective reactions of the dipole. Habituating chemical transmitters within a gated dipole determine an affective adaptation level, or context, against which later events are evaluated. Neutral events can become affectively charged either through direct activations or antagonistic rebounds within a previously habituated dipole. The theory describes the affective consequences of strategies in which an individual compares pairs of events or statements that are not necessarily explicitly grouped within the stimuli. The same preference orders may sometimes, but not always, emerge from different sequences of pair-wise alternatives. The role of short-term memory updating and expectancy-modulated matching processes in regulating affective reactions is described. The formal axioms of prospect theory are dynamically explicated through this analysis. Analyses of judgments of the utility of a single alternative, choices between pairs of regular alternatives, choices between riskless and risky alternatives, and choices between pairs of risky alternatives lead to explanations of such phenomena as preference reversals, the gambler's fallacy, the framing effect, and the tendency toward risk aversion when gains are involved but risk taking when losses are involved. These explanations illustrate that data concerning decision making under risk may now be related to data concerning the dynamics of conditioning, cognition, and emotion as consequences of a single psychophysiological theory.

1. Some Previous Models of Risky Decision Making

Most environments are characterized by some degree of uncertainty. Environmental uncertainty may be inherent in some situations, such as a coin toss, or may arise due to imperfect information about the physical or social environment. Whatever the source, analysis of the structural and functional characteristics of uncertainty is a problem of considerable importance in a wide variety of research areas including mathematics, economics, and psychology.

The multidisciplinary approach to the study of decision making under risk has led to the development of two distinct types of theory: normative and descriptive. Normative theories are prescriptive in nature because they are concerned with devising decision-making procedures or algorithms that are optimal with regard to some set of intuitively reasonable constraints. Descriptive theories, on the other hand, are concerned with providing an accurate portrayal of how individuals actually make decisions, independent of whether those decisions are optimal or even logical.

Historically, the distinction between the two types of theory has been blurred because normative theories, particularly expected utility theory, have been widely accepted as adequate descriptions of how individuals integrate information when making risky decisions. For example, the earliest form of utility theory, which was developed by Daniel Bernoulli as a solution to the gambling puzzle known as the St. Petersburg Paradox, was very influential in economics for over a century (Bernoulli, 1738). More recently, the axiomatic form of utility theory, which was first developed by von Neumann and Morgenstern (1944), has been assumed to provide an acceptable descriptive model of decision making under risk.

Since the introduction of axiomatic utility theory, a large body of evidence has accumulated which demonstrates that individuals systematically violate some of the fundamental tenets of rational choice (e.g., Allais, 1953; Tversky & Kahneman, 1981). These numerous violations of rationality have motivated a great deal of experimental and theoretical work aimed at developing a more accurate descriptive theory of decision making under risk than is provided by utility theory. Overall, these

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efforts have been quite successful in the sense that many violations of the axioms of utility theory and rational choice are explicable within one or more of these theoretical frameworks. For example, violations of the betweeness axiom of utility theory are consistent with the axiomatic framework of portfolio theory (Coombs, 1975).

The particular theory that has been the most successful in uncovering and explaining these violations is prospect theory (Kahneman & Tversky, 1979). This theory adopts a number of coding, psychophysical, and decision-rule assumptions that provide a natural account of many results that are inconsistent with the axioms of utility theory and general assumptions of rational choice. To illustrate, prospect theory explains the reflection effect, in which choices involving gains tend to be risk averse, whereas choices involving losses tend to be risk taking. It does so using an S-shaped value function and a decision weight function that is characterized by the properties of subcertainty and subproportionality (Section 7). These psychophysical assumptions, in combination with certain coding assumptions, also provide a plausible explanation of framing effects that involve shifts in preference depending on whether the outcomes of choices are stated positively or negatively (Section 13).

Despite the impressive array of data that are accounted for by prospect theory, the theory is not immune to criticism. One criticism is that prospect theory is a static, algebraic theory that relies heavily on psychophysical functions derived from analyses of group choice data. As a result, the theory provides little insight into the information-processing dynamics that underlie risky decision making. A second and perhaps sharper criticism of prospect theory is that it does not account for all important forms of nonrational decision making. The problem revolves around the decision rule of prospect theory which assumes that individuals act to maximize subjective value. A vivid illustration of the difficulty with this seemingly compelling assumption is the paradoxical result that is called the preference reversal phenomenon. Preference reversals are observed when, in a binary choice situation, an individual prefers an alternative that has been judged to be worth less than the nonpreferred alternative. To illustrate, an individual might judge one alternative to be worth $10 when presented in isolation and a second alternative to be worth $8 when presented in isolation and yet prefer the second alternative when given a choice between the two alternatives. It is important to note that the preference reversal phenomenon is a robust effect that cannot be dismissed as statistical noise, at least for the theoretically interesting cases where the alternatives are reasonably close in value (Grether & Plott, 1979; Gutowski & Chechile, 1984; Hamm, 1979; Lichtenstein & Slovic, 1971, 1973; Lindman, 1971; Mowen & Gentry, 1980; Pommerehne, Schneider, & Zweifel, 1982; Reilly, 1982; Slovic & Lichtenstein, 1983). Therefore, preference reversals violate the maximization assumption of prospect theory and constitute an important and as yet inadequately explained example of nonrational choice.

This is not to say that the preference reversal phenomenon has escaped theoretical attention. Lichtenstein and Slovic (1971), for example, assumed that judgment and choice task requirements generate different information-processing sequences. More specifically, their formulation assumes that judgments of the subjective value of risky alternatives follow an anchoring-and-adjustment process, in which either the amount that can be won or lost serves as the anchor and choice is primarily governed by the probability of winning. Although such a differential weighting model can account for much of the data on preference reversals (for an exception see Gutowski & Chechile, 1984), the model in its present form is not sufficiently general to predict other phenomena in the domain of decision making under risk.

In this article we describe an alternative theory of decision making under risk, one that we call affective balance theory. The theory is best viewed as an application of a more general theory of how cognitive and emotional processes interact (Grossberg, 1980, 1982a, 1984b, 1987). This general theory has been used to explain phenomena in such diverse areas as perception, attention, motivation, learning, and memory. Affective balance theory uses psychophysiological mechanisms and processes that have been derived from data analyses in these other areas to build a dynamic description of the affective and cognitive events that underlie risky judgment and choice. The theory clarifies how context affects the processing of risky information and provides an explanation of a number of well-established phenomena in risky decision making, including preference reversals. The present work thus suggests how properties of decision making under risk may be explained as manifestations of a processing theory that was developed to explain a quite different data base. This linkage relates phenomena concerning human decision making under risk to human evoked potentials, neurophysiological and pharmacological substrates of behavior, animal discrimination learning, human memory and attentional processing, and certain mental disorders. In particular, the arguments developed herein apply the same mechanisms that have previously been used to analyze such phenomena as hypothalamic self-stimulation, secondary conditioning, asymptotically nonchalant avoidance behavior, conditioned emotional responses, superconditioning, analgesia, differential rewarding effects of sudden versus gradual shocks, self-punitive behavior, and learned helplessness (Grossberg, 1971, 1972a, 1972b), as well as the partial reinforcement acquisition effect, behavioral contrast, blocking and unblocking, schedule-induced polydipsia, rebound eating, intragastric drinking, hyperphagic eating, the Valenstein effect, and latent inhibition (Grossberg, 1975, 1982a, 1984b, 1987). Within the broader context of animal discrimination learning and choice behavior, all of these phenomena may be viewed as variants of decision making under risk.

Affective balance theory generates formal relationships that have been posited by algebraic models of decision making under risk in the form of emergent properties of real-time circuits that have been used to analyze and predict a wide range of interdisciplinary psychophysiological data. In this sense, affective balance theory "explains" these formal relationships. In so doing, as an automatic consequence, it also explains the data that these formal relationships have been used to fit. In addition, affective balance theory predicts a number of other data—for example, data concerning preference reversals (Section 10)—that cannot easily be accounted for by a number of other theories, notably prospect theory.

Rachlin, Logue, Gibbon, and Frankel (1986) have also realized the importance of concepts about instrumental behavior for understanding decision making under risk. These authors applied algebraic form factors that have been developed to fit instrumental data collected from animals to analyze human data about decision making under risk. The present article develops a real-time neural network model, rather than algebraic form factors, and thereby provides a mechanistic analysis of decision-making data in the same way that this model has elsewhere suggested a mechanistic analysis of a large literature about instrumental behavior.

The present theory does not attempt to characterize all of the possible cognitive strategies that individuals may invoke. Rather, it analyzes the effects of a chosen strategy on the affective values of the events to which an individual is exposed. In particular, the theory analyzes the affective consequences of strategies in which a subject compares pairs of events or statements that are not necessarily explicitly grouped within the stimulus materials. The cognitive context in which individual events are embedded may alter the comparisons that a subject makes to arrive at a preference order for these events. In Section 8, we note that the same preference order can sometimes emerge from different sequences of pair-wise comparisons. This result brings into a sharper focus those circumstances wherein preference order does depend on the sequencing of event comparisons. In addition, in Section 12, we show how the cognitive context with which a neutral event, such as a zero outcome, is compared can endow the event with a positive or a negative affect, depending on the comparisons afforded by the context. In Section 13, we discuss how the cognitive context can influence, or "frame," the set of outcomes that the subject will be inclined to process, including the pair-wise comparisons that the subject constructs from the stimulus materials.

Within affective balance theory, affective or emotional processing is assumed to be regulated by an opponent process that is called a *gated dipole* (Grossberg, 1972a, 1972b, 1984a, 1984b). Four main ingredients go into the design of a gated dipole: slowly accumulating chemical transmitters that are designed to generate unbiased transductions of their inputs; opponent, or competitive, interactions between an on-channel and an off-channel; phasic inputs that perturb the on-channel or the off-channel through time; and a sustained, or tonic, arousal level that equally perturbs both channels and thereby sets the sensitivity of dipole outputs to phasic input fluctuations.

A number of other opponent processing models exist in the literature. Jensen (1970, 1971) described qualitatively some of the properties that a good theory of opponent processing should have and applied these properties to the analysis of conditioning data. Solomon and Corbit (1974) and Solomon (1980, 1982) also used opponent processing ideas to analyze data about conditioning and, more generally, affective processing. A comparison of the gated dipole opponent process and the type of opponent process described by Solomon and Corbit (1974) is given in Section 5. We now summarize the gated dipole properties that we will need.

2. Transmitter Gates: Unbiased Transmitter-Modulated Signaling

The simplest rule whereby one nerve cell site can send unbiased signals to another nerve cell site is as follows. If $S(t)$ is the input signal to one cell site and $T(t)$ is the output signal to the next cell site, then the linear relationship

$$T = SB, \tag{1}$$

where B is a positive constant, is the simplest law of unbiased transmission. By Equation 1, the outgoing signal is proportional to the incoming signal, and the signal is relayed perfectly.

When the output signal $T(t)$ is due to the release or inactivation of a chemical transmitter $z(t)$ in response to the input signal $S(t)$, further consideration is necessary. How is a large and sustained input $S(t)$ prevented from depleting $z(t)$ and thereby causing a progressively smaller signal $T(t)$? In other words, when $T(t)$ is due to the release or inactivation of a transmitter, the term B in Equation 1 may not be constant. It may decrease through time as $z(t)$ is depleted, thereby reducing the sensitivity of $T(t)$ to $S(t)$. In this situation, Equation 1 is replaced by the equation

$$T = Sz. \tag{2}$$

Our task is to analyze how $z(t)$ approximates a constant B,

$$z \cong B, \tag{3}$$

despite its depletion due to input S.

Equation 2 says that transmitter z is released or inactivated at a rate (proportional to) T in response to input S. In other words, z *gates* S to generate T, or T is caused by a *mass action* interaction between S and z. By Equation 2, an increase in either S or z can increase T, and no output signal T can be generated if either no input signal occurs ($S = 0$) or no transmitter is available ($z = 0$).

Equation 3 requires that the *sensitivity* of T to S be maintained through time. If both Equations 2 and 3 are simultaneously implemented, as in Equation 1, then unbiased transmission by a depletable chemical is achieved. Equation 1 means that $z(t)$ is replenished instantaneously, or at least at a rate that is rapid relative to the rate of gated release or inactivation. In our applications, the rate of accumulation is slow relative to the rate of gated release or inactivation. In order to represent this type of process, an algebraic equation is insufficient. A differential equation is needed. We use the simplest differential equation that is capable of reconciling Equations 2 and 3 when both the accumulation and gating processes take place at a finite rate relative to the rate with which the signal S can fluctuate. In this situation, Equations 2 and 3 are not both exactly satisfied at any one time. The process attempts to achieve unbiased transmission but can do so only approximately due to its finite reaction rates. Such a slow-down of transmitter accumulation does not reflect a system failure. It provides the basis for conditioning properties of fundamental importance (Grossberg, 1972a, 1972b, 1982a, 1987). Thus we view the slow rate of transmitter accumulation as an evolutionary specialization that has persisted due to its behavioral value. Here we suggest that it also underlies several basic properties of decision making under risk.

The simplest differential equation capable of simultaneously implementing Equations 2 and 3 is the following one (Grossberg, 1972b):

$$\frac{d}{dt} z = A(B - z) - CSz, \tag{4}$$

where A, B, and C are positive. In Equation 4, the notation $d/dt\ z$ denotes the net production rate of z. Term $A(B - z)$ says that z accumulates at a rate A until it reaches the target level B, as required by Equation 3. Term $-CSz$ says that the loss of transmitter per unit time due to gating is proportional to Sz, as required by Equation 2. Henceforth we choose $C = 1$ for notational simplicity. This amounts to rescaling the size of S.

Term $A(B - z)$ may be physically instantiated in more than one way. For example, a passive accumulation of z may occur onto unoccupied sites whose total number is B. Alternatively, transmitter precursors may actively be produced at a rate AB, but feedback inhibition via term $-Az$ of transmitter z onto an intermediate stage of production may reduce the net production level to $A(B - z)$. Without such feedback inhibition, transmitter production would continue unabated until the cell ruptured.

In response to a constant signal of size S, Equation 4 implies that the transmitter z approaches the equilibrium value

$$z = \frac{AB}{A + S}. \tag{5}$$

In other words, larger signals S deactivate more transmitter. On the other hand, the output signal that is generated by an input S does not equal z. The output signal is equal to $T = Sz$, due to Equation 2.

Figure 1 describes how the output T reacts to changes in the size of the input S. A rapid increase in S from S_0 to S_1 elicits a slow decrease in z. Multiplication of the graphs of $S(t)$ and $z(t)$ shows that a rapid increase in S generates a rapid increase in T followed by a slow decrease, or habituation, of T to an intermediate level. In a similar way, a rapid decrease in S from S_1 to S_0 generates a rapid decrease in T followed by a slow increase, or habituation, to an intermediate level. In all, rapid increases and decreases in the input S generate overshoots and undershoots in the output T due to the slow rate of reaction, or habituation, of the transmitter as it seeks to generate unbiased signals. These habituative reactions are fundamental to many of the explanations given by our theory.

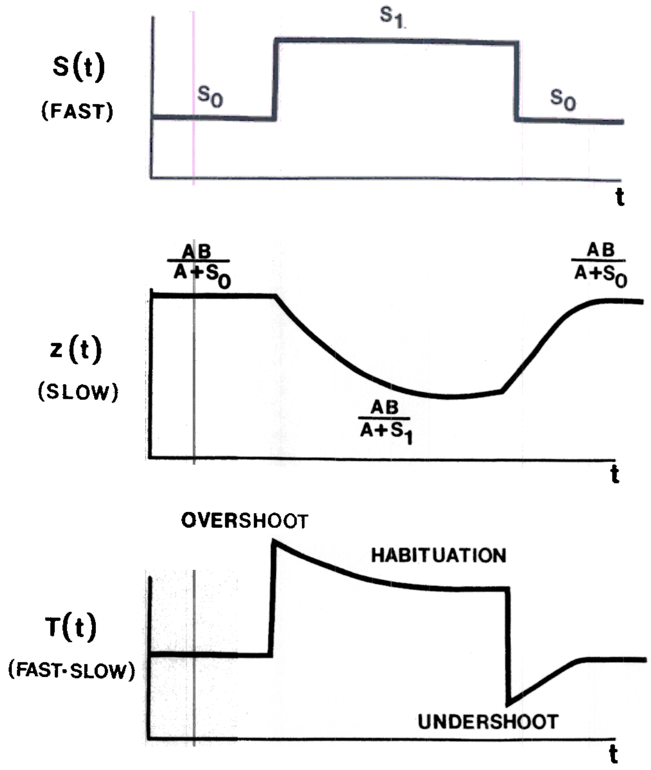

Figure 1. Reaction of output signal T and transmitter gate z to changes in input S. (The output T is the product of a fast process S and slow process z. Overshoots and undershoots in T are caused by z's slow habituation to fast changes in S.)

3. Gated Dipoles: Tonically Aroused Transmitter Gates in Opponent Processes

Figure 2 describes one of the basic properties of a gated dipole. In such an opponent process, a phasic input (J) can elicit a sustained on-response, whereas offset of the input can elicit a transient off-rebound, or temporal contrast effect. These properties are explained as follows.

The left-hand series of stages in Figure 2 represents the on-channel, and the right-hand series of stages represents the off-channel. Both channels receive an equal arousal input, denoted by I, that is constant through time. The arousal input energizes the antagonistic rebound that occurs after the on-input shuts off. The on-input, denoted by J, is delivered only to the on-channel. Input J is switched from zero to a positive level and held at that level long enough for gate equilibration to occur. Then J is shut off.

Inputs I and J are added by the activity (or potential) $x_1(t)$. Activity $x_1(t)$ responds quickly to input fluctuations, relative to the reaction rate of the network's slow transmitter gates. The graph of $x_1(t)$ has the same form as the top graph in Figure 1: a rapid switch from a lower positive activity to a higher positive activity, followed by a rapid return to the lower level. The activity $x_1(t)$ generates an output signal $f(x_1(t))$ in its pathway that again has the form of a double-switch between two positive values. The output signal $f(x_1(t))$ is gated by a slow transmitter $z_1(t)$ that accumulates and is inactivated from the square synapse in the on-channel. Figure 1 describes the effect of this slow gate on the input to the next stage. Consequently, activity $x_3(t)$ follows an overshoot–habituation–undershoot–habituation sequence through time. Then $x_3(t)$ relays an output signal of the same form to $x_5(t)$. Activity $x_5(t)$ also receives an inhibitory signal from $x_4(t)$. To determine what happens next, we consider the dynamics of the off-channel.

The off-channel receives only the constant tonic input I. Hence $x_2(t)$ and the slow gate $z_2(t)$ in the off-channel square synapses are constant through time. The activity $x_4(t)$ is therefore also constant through time. For definiteness, we make the simplest assumption that corresponding stages in the on-channel and the off-channel possess the same parameters. Because the arousal input I to both channels is also equal, the size of x_4 equals the baseline activity level of $x_3(t)$. This is not always true, but its violation is easy to analyze after the symmetric case is understood.

We can now determine the reactions of activity $x_5(t)$ through

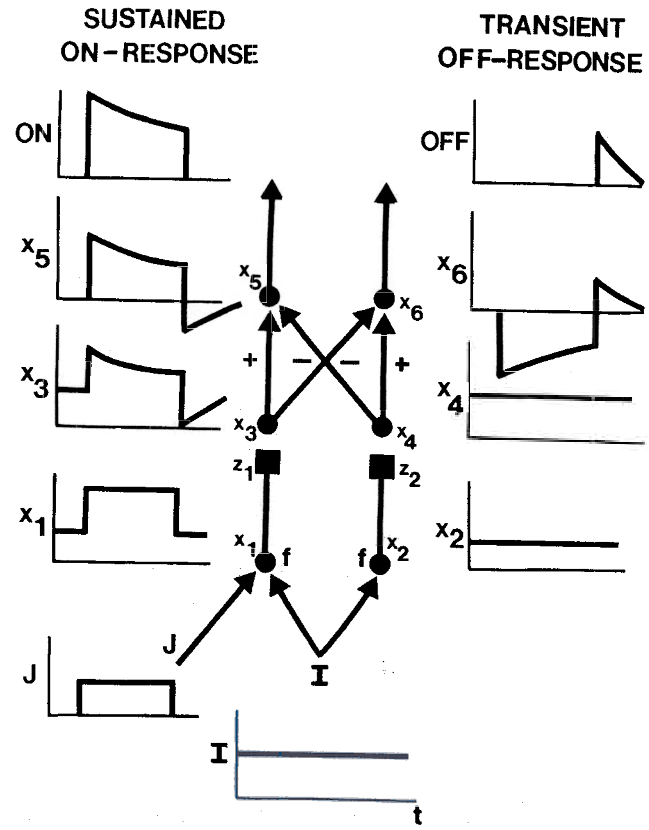

Figure 2. Example of a gated dipole. (A sustained habituating on-response [top left] and a transient off-rebound [top right] are elicited in response to onset and offset, respectively, of a phasic input J [bottom left] when tonic arousal I [bottom center] and opponent processing [diagonal pathways] supplement the slow gating actions [square synapses]. See text for details.)

time. Because the signals from $x_3(t)$ and $x_4(t)$ subtract before perturbing $x_5(t)$, and their baseline activities are the same, the baseline activity of $x_5(t)$ equals zero. Activity $x_5(t)$ thus overshoots and undershoots a zero baseline when the input J is turned on and off. By contrast, activity $x_6(t)$ responds in an opposite way from $x_5(t)$ because x_3 excites x_5 and inhibits x_6, whereas x_4 inhibits x_5 and excites x_6.

The final assumption is that the output signals caused by activities $x_5(t)$ and $x_6(t)$ are rectified: Outputs are generated only if these activities exceed a nonnegative threshold. As a result, the on-channel generates a sustained output signal while the input J is on. This output signal habituates as the gate $z_1(t)$ slowly equilibrates to the input. By contrast, the off-channel generates a transient off-response, or antagonistic rebound, after the input J shuts off.

4. Mathematical Properties Leading to Antagonistic Rebound

We now describe the simplest formulas that can instantiate Figure 2 in order to set the stage for our computations about risky decision making. Let the total signal in the on-channel be $S_1 = f(I + J_1)$ and the total signal in the off-channel be $S_2 = f(I + J_2)$, where I is the baseline level of arousal that perturbs both channels and f is a function that transforms these inputs into signals. In general, f is a nonnegative and monotone increasing function such that $f(0) = 0$. Assume that $J_1 > J_2$, and hence that $S_1 > S_2$. As in Equation 4, let the transmitter in the on-channel, z_1, satisfy the equation

$$\frac{d}{dt} z_1 = A(B - z_1) - S_1 z_1$$

and the transmitter in the off-channel, z_2, satisfy the equation

$$\frac{d}{dt} z_2 = A(B - z_2) - S_2 z_2 \tag{7}$$

Assume that the inputs S_1 and S_2 are present for a sufficient amount of time so that z_1 and z_2 equilibrate, or habituate, to S_1 and S_2. At equilibrium, $d/dt\ z_1 = d/dt\ z_2 = 0$. Thus, by Equations 6 and 7

$$z_1 = \frac{AB}{A + S_1} \tag{8}$$

and

$$z_2 = \frac{AB}{A + S_2} \tag{9}$$

By Equation 2, the gated signal in the on-channel is then

$$T_1 = S_1 z_1 = \frac{ABS_1}{A + S_1},$$

and the gated signal in the off-channel is

$$T_2 = S_2 z_2 = \frac{ABS_2}{A + S_2}. \tag{11}$$

After the two channels compete, the net activity in the on-channel is

$$\begin{aligned} x_5 &= T_1 - T_2 \\ &= S_1 z_1 - S_2 z_2 \\ &= \frac{A^2 B(S_1 - S_2)}{(A + S_1)(A + S_2)} \end{aligned}$$

Note that this on-activity is positive because $S_1 > S_2$. By contrast, the off-activity is negative because $x_6 = -x_5$ (Figure 2). After the thresholds act, the on-output is positive whereas the off-output equals zero.

In order to understand how off-rebounds occur, eliminate the inputs J_1 and J_2. The inputs to each channel then both equal I. However, the transmitters z_1 and z_2 are assumed to change slowly, so that Equations 8 and 9 are approximately valid for some time interval after the offset of J_1 and J_2. The gated signals during this interval are then approximately

$$T_1 = f(I) z_1 = \frac{ABf(I)}{A + S_1} \tag{13}$$

and

$$T_2 = f(I)z_2 = \frac{ABf(I)}{A + S_2} \quad (14)$$

After competition, the net off-rebound is

$$x_6 = f(I)z_2 - f(I)z_1 = \frac{ABf(I)(S_1 - S_2)}{(A + S_1)(A + S_2)}, \quad (15)$$

which is positive, whereas the net on-response is $x_5 = -x_6 < 0$. Owing to the output threshold, an antagonistic rebound, or contrast effect, occurs.

The rebound is transient because the transmitters z_1 and z_2 both equilibrate in response to I and approach levels $AB(A + I)^{-1}$. As a result, x_6 approaches zero. Thus the competition between on-channel and off-channel eventually shuts off both channels when they receive equal inputs for a sufficient amount of time for equilibration to occur.

5. Comparison With the Solomon and Corbit Opponent Process Model

The antagonistic rebound in the off-channel of a gated dipole is energized by an undershoot of the dipole's on-activity function x_3 (Figure 2). In a gated dipole, such an undershoot is due to habituation of the transmitter gate within the on-channel. Overshoots and undershoots have also been hypothesized to exist in alternative models of opponent processing, but the properties have not been traced to the action of a slowly habituating transmitter gate. For example, Solomon and Corbit (1974) and Solomon (1980, 1982) described a model of opponent processing in which overshoots and undershoots occur. These authors ascribed the overshoots and undershoots to the subtraction of two opponent processes that both evolve according to similar time scales (Figure 3). Neither process, in itself, undergoes an overshoot or an undershoot. Instead, overshoots and undershoots are derived from the assumption that the off-process begins to build up only after the on-process is initiated. The model assumes, in addition, that "the second component, the *b* process, is aroused via the arousal of *a*" (Solomon & Corbit, 1974, p. 126). Neither assumption is made in a gated dipole opponent process, wherein the slow habituation of the transmitter gate within the on-channel generates an overshoot and an undershoot within that channel. Consequently, in a gated dipole, opponent processing per se between the on-channel and the off-channel generates the antagonistic rebound within the off-channel without necessitating the hypothesis that on-channel activation triggers a delayed off-channel activation.

The hypotheses of the Solomon and Corbit model may be challenged on several fronts. The hypothesis of delayed activation of the off-channel by the on-channel seems problematic when one asks how a direct activation of the off-channel can cause a delayed activation of the on-channel. Solomon and Corbit did not raise this question. Indeed, they did not separate the on- and off-components into two topographically distinct output pathways. More generally, their opponent process is not defined by a dynamical model. Instead, their components were chosen to fit the data in different experimental paradigms. For example, the Solomon and Corbit model does not explain why

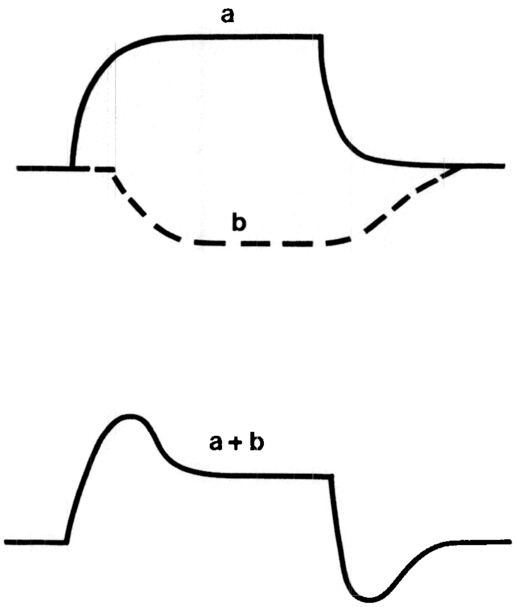

Figure 3. The opponent process model of Solomon (1982), in which overshoots and undershoots are caused by an excitatory process *a* and an inhibitory process *b* that both change at a similar rate such that *b* lags behind *a* and neither *a* nor *b* separately exhibits overshoots or undershoots.

the maximum size of the *a* process should sometimes, but not always, exceed the maximum size of the *b* process, or why the *b* process is delayed in time relative to the *a* process by just the right amount to produce an overshoot and an undershoot. The hypothesis that slowly habituating, tonically aroused, transmitter gates exist in an opponent anatomy provides simple answers to all of these questions and implies other properties that enable the gated dipole model to organize data about decision making under risk.

6. The Psychophysics of Risk: Short-Term Memory and Long-Term Memory Interactions

The final set of assumptions of the affective balance theory may be thought of as describing the psychophysics of risk. More specifically, these assumptions describe how events, notably probabilistically experienced events, are transformed and encoded. Although the resulting algebraic properties of these assumptions are quite similar to those adopted by other theories of decision making under risk (see below for a comparison with prospect theory), the rationale for these assumptions is quite distinct. The properties here are based on analyses of how environmental events are coded into internal representations in order to solve inevitable dilemmas posed by a fluctuating and uncertain environment. As a result, the theory provides a dynamic, rather than static or formal, description of the psychophysics of risk. Further, the theory strengthens the rationale for these assumptions by extending the data base that motivates these assumptions beyond group choice data from studies of risky decision making. In the following paragraphs, we briefly summarize the main processing ideas about the psychophysics of risk that motivate our computations of risky decisions. We do not redevelop these processing ideas here but refer the reader to the original sources. Our summary is intended to provide meaning to the computations, which themselves form the core of the present contribution.

The analysis of how probability information has an impact on risky decisions begins with the observation that the storage of individual events and their associated affective values in short-term memory is positively related to the frequency or probability of occurrence. This property follows from two interacting effects, one long term and the other short term. The long-term effect concerns the greater influence of familiar events than of unfamiliar events on tuning of the long-term memory traces that regulate coding of an event in short-term memory (Grossberg, 1980; Grossberg & Stone, 1986). Other things being equal, a better match of an event with the patterning of long-term memory traces leads to enhanced activation in short-term memory of the event's internal representation. Thus, to the extent that the chosen high-probability events are more familiar than the chosen low-probability events, the short-term memory activity of an event will tend to be an increasing function of its prior probability. However, the total effect of probability is more complicated, especially when both low-probability and high-probability events are equally familiar.

This is because the short-term effect of probability also influences the action of cognitive expectancies. In particular, low-probability events tend to be more unexpected than high-probability events. They can therefore trigger a more complete reset of prior short-term memory, thereby facilitating their own preferential loading into short-term memory (Grossberg, 1982b, 1987). The net effect of this short-term effect is often that the ratio of the short-term memory activity of a low-probability event to that of a high-probability event is greater than the ratio of their respective probabilities. If we assume that the "decision weight" associated with a probabilistic event is the average short-term memory activity across events with that particular probability, then we are led to the assumption that low-probability events will be overweighted relative to high-probability events.

Our analysis of how value or utility information affects risky decision making begins with the observation that all neural signal functions must be bounded. Consequently, the value function will be chosen to be slower-than-linear or negatively accelerated at large values of both positively and negatively valenced events. It has, moreover, been shown mathematically that the simplest bounded function capable of transducing neural activities into signals, without amplifying noise, is an S-shaped, or sigmoid, function (Grossberg, 1973, 1983; Grossberg & Levine, 1975). Using this sigmoid function as a starting point, the value function is computed as follows for gains and losses. It is assumed that the value, or affective magnitude, of a gain is a function of the on-response to that event, whereas the value of a loss of a given magnitude is a function of the antagonistic rebound that occurs in response to the removal of a positive event of that magnitude. If we divide Equation 15 by Equation 12, we see that the size of the antagonistic rebound relative to the on-response is simply $f(I)/A$, which is a function only of A and I. More specifically, the antagonistic rebound is larger than the on-response whenever $f(I)$ is larger than A. Because this inequality

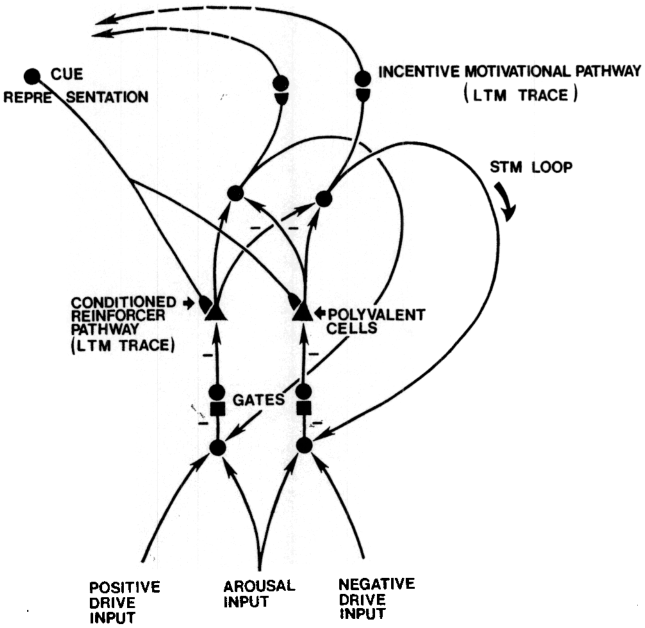

Figure 4. A cognitive-emotional interaction between cognitive, or cue, representations and a gated dipole opponent process, called a drive representation, that synthesizes reinforcing signals, internal drive inputs, and an arousal baseline into an affective response. (This response generates incentive motivational feedback signals to the cue representations and can thereby cause a shift in attention toward motivationally salient information. Signals from cue representations to drive representations are multiplied by long-term memory [LTM] traces that encode the conditioned reinforcing values of the cue representation. Unconditioned reinforcers can also activate the drive representation. [From "Processing of Expected and Unexpected Events During Conditioning and Attention: A Psychophysical Theory" by S. Grossberg, 1982, *Psychological Review, 89,* p. 551. Copyright 1982 by the American Psychological Association, Inc. Reprinted by permission.])

ordinarily holds in individuals capable of learned avoidance behavior (Grossberg, 1972b), we are led to conclude that the overall value function is not symmetric but rather is usually steeper for losses than gains. This analysis leads to the interesting and testable implication that certain underaroused individuals, for whom I is pathologically small, may show the opposite pattern (Grossberg, 1984a, 1984b).

The final psychophysical issue concerns how probability and outcome information combine or interact. In Grossberg's theory of cognitive-emotional interactions, activation of an event's short-term memory representation elicits signals from this representation to the gated dipole opponent processes where emotional reactions are generated (Figure 4). Before these signals can reach the gated dipoles, they are multiplied, or *gated,* by long-term memory traces that encode the conditioned reinforcer values of the event. These gated signals then activate the on-channel or the off-channel of their target dipoles (Grossberg, 1972b, 1987), thereby leading to emotional reactions and motivational signals. In other words, the net affective activity associated with an event of a given probability is the product of the (expectancy-modulated) short-term memory strength of that event and the affective value of that event as read out of long-term memory into short-term memory.

In order to facilitate comparisons with other theories of decision making under risk, we provide a more formal summary of the main psychophysical properties of the theory. The theory assumes that each (affectively meaningful) dipole input, J^+ and J^-, is the product of two factors. The first factor, which we denote by $f(x)$, is a signal readout from the short-term memory representation x of each event. The second factor, $z^+(x)$ or $z^-(x)$, is a long-term memory trace that encodes the conditioned reinforcing value, positive or negative, of an event. For positively valenced events the signal to the on-channel may then be expressed as

$$J^+(x) = f(x)z^+(x) \tag{16}$$

due to the gating of the short-term memory signal $f(x)$ by the long-term memory trace $z^+(x)$. Similarly, the gated signal of a negatively valenced event to the off-channel may be expressed as

$$J^-(x) = f(x)z^-(x). \tag{17}$$

The theory assumes that the function $f(x)$ is characterized by the psychometric property termed *subproportionality,* because the ratio of the expectancy-modulated short-term memory activity of a low-probability event to that of a high-probability event is ordinarily greater than the ratio of their respective probabilities (Figure 5a). Finally, the functions $z^+(x)$ and $z^-(x)$ are S-shaped about the origin (or status quo), with the long-term memory (LTM) trace $z^-(x)$ of negatively valenced events steeper than the LTM trace $z^+(x)$ for positively valenced events (Figure 5b) due to the antagonistic rebound properties cited above.

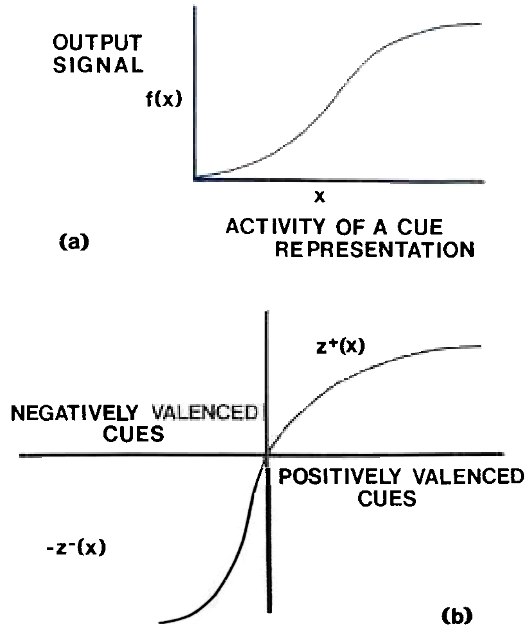

Figure 5. Relationship of dynamic neural processes to algebraic psychophysical processes: (a) A sigmoid signal function helps to achieve subproportionality, as does the modulation of short-term memory by matching with learned expectations; (b) the long-term memory traces $z^+(x)$ and $z^-(x)$, which input to, and learn from, the positive and negative channels, respectively, of an affective gated dipole, derive their shape from direct on-reactions to inputs as well as antagonistic rebound off-reactions to changes in these inputs.

7. Comparison With Prospect Theory

We now compare the psychophysical properties assumed in affective balance theory with the psychophysical assumptions of prospect theory (Kahneman & Tversky, 1979). This summary does not provide a complete characterization of prospect theory, but rather focuses on the assumptions of that theory that govern the psychophysics of risk.

The first psychophysical assumption of prospect theory involves a scale, $v(x)$, that describes how an event x is transformed into a subjective value. The theory assumes that the value function is normally concave for positive changes in status or gains but is normally convex for negative changes or losses. That is, the marginal value of both gains and losses generally decreases with larger magnitudes. The theory further assumes that the value function for losses is generally steeper than the value function for gains. A hypothetical value function that meets these criteria is shown in Figure 6a.

The second psychophysical assumption involves a scale, $\pi(P)$, that describes how a probability P is transformed into a decision weight that calibrates the relative impact of an event with a particular probability on the overall value of a risky alternative. The theory naturally assumes that the weighting function, $\pi(P)$, is a monotonically increasing function of P with $\pi(0) = 0$ and $\pi(1) = 1$. In addition, it is assumed that small probabilities are generally overweighted but that large probabilities are generally underweighted. More specifically, the theory assumes a property called *subcertainty* so that

$$\pi(P) + \pi(1 - P) < 1 \tag{18}$$

for all $0 < P < 1$. Prospect theory also assumes the property of *subproportionality* that was discussed earlier. Mathematically, subproportionality can be expressed as

$$\frac{\pi(Pq)}{\pi(P)} < \frac{\pi(Pqr)}{\pi(Pr)} \tag{19}$$

for all $0 < r < 1$. That is, for a fixed ratio of probabilities, the ratio of decision weights is closer to unity for small probabilities than for large probabilities. A weighting function that meets these criteria is shown in Figure 6b.

The third psychophysical assumption is an integration rule that describes how probability and event value information combine. Prospect theory assumes that this integration rule is multiplicative, so that the net contribution of an event of a particular probability to the overall value of an alternative is $\pi(P)v(x)$.

On comparing the psychometric assumptions of affective balance theory and prospect theory, we find three obvious points of correspondence. First, the value function, $v(x)$, in prospect theory and the long-term memory functions, $z^+(x)$ and $z^-(x)$, in affective balance theory both represent the positive or negative value associated with previously experienced events. Moreover,

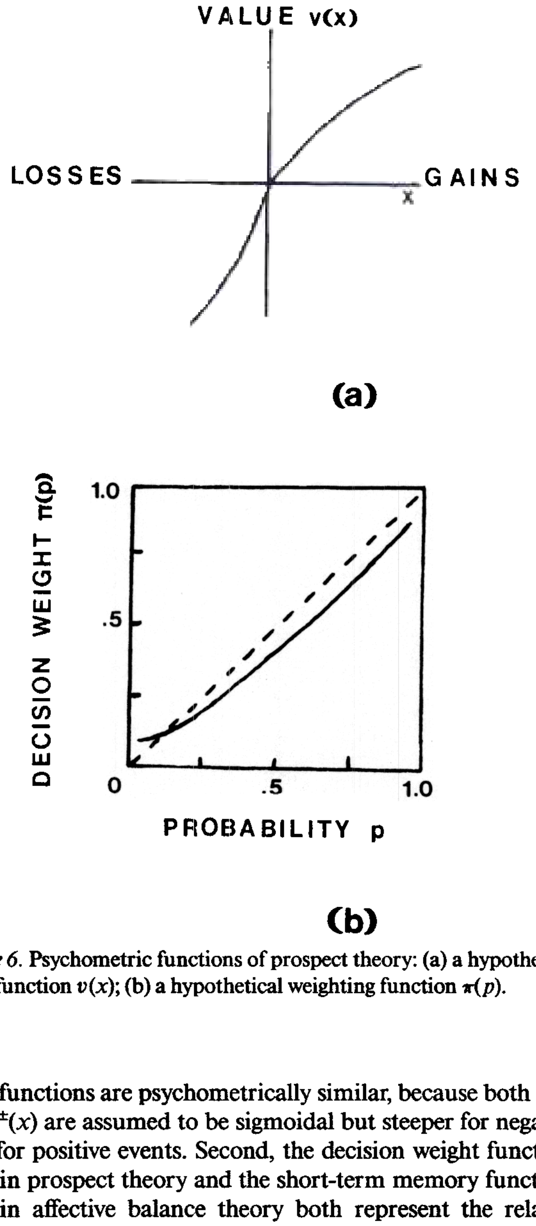

Figure 6. Psychometric functions of prospect theory: (a) a hypothetical value function $v(x)$; (b) a hypothetical weighting function $\pi(p)$.

these functions are psychometrically similar, because both $v(x)$ and $z^{\pm}(x)$ are assumed to be sigmoidal but steeper for negative than for positive events. Second, the decision weight function, $\pi(P)$, in prospect theory and the short-term memory function, $f(x)$, in affective balance theory both represent the relative strength of an event with a particular probability in active memory. These functions also are characterized by similar psychometric properties such as subproportionality. Third, the integration rule, $\pi(P)v(x)$, in prospect theory and the gating law, $J(x) = f(x)z(x)$, in affective balance theory both assert that probability and event value information combine multiplicatively.

These three points of correspondence illustrate our assertion that the psychophysical properties of affective balance theory are similar to those of other prominent theories of decision making under risk. In addition, affective balance theory provides a description of the information-processing dynamics that underlie these psychophysical properties. In particular, the short-term memory function $f(x)$ simultaneously reads out $J^+(x) = f(x)z^+(x)$ and $J^-(x) = f(x)z^-(x)$ into the opponent process of the gated dipole. This opponent process sometimes generates the net reaction $J^+(x) - J^-(x) = f(x)[z^+(x) - z^-(x)]$ to these inputs, thereby rationalizing a comparison between the value function $v(x)$ of prospect theory and the difference $z^+(x) - z^-(x)$ of the LTM traces. However, the gated dipole opponent process often does *not* merely subtract the $z^+(x)$ and $z^-(x)$ LTM functions. These deviations from additivity are, moreover, a principal source of the gated dipole's ability to explain difficult data about affectively charged behaviors.

8. The Temporal Unfolding of Risky Judgments: Generating an Affective Context

We now illustrate how different temporal sequences of event comparisons control the affective values of these events. First we consider a temporal sequence used to judge the value of a two-outcome regular alternative. Then we consider two different event sequences that may occur under conditions of risky choice and show how both sequences can generate the same preference order.

We first observe that judgments of a two-outcome regular alternative may initially be unorderly (i.e., not monotonic in the outcome and probability variables) and require considerable practice before stabilizing (Anderson & Shanteau, 1970). The model assumes that this transition or learning phase includes development of a stable adaptation level against which alternatives may be evaluated or processed. Although no precise assumptions are made about the adaptation level, J_0, it is assumed that J_0 is a function of the previous inputs and that it perturbs both channels equally. This assumpion may be relaxed when the average magnitudes of previous positive and negative inputs are not approximately equal.

The theory assumes that the judgment process begins by setting the adaptation level at J_0. Instatement of J_0 drives the transmitter levels in each channel of the dipole to levels $z_0 = AB/[A + f(I + J_0)]$. (Henceforth we assume for simplicity that J_0 is absorbed into the arousal level I and that the theory is operating in the linear range of f: $f(w) = w$.) After the channels adapt, it is assumed that attention is directed to the current alternative a_i. The positive and negative inputs to the on-channel and off-channel are then J_i^+ and J_i^-, respectively. However, the transmitter levels change slowly so that the levels z_0 are maintained (approximately) for an interval after J_i^+ and J_i^- are presented. After transmitter gating and opponent processing, the dipole on-response to a_i is given by

$$r_i = (I + J_i^+)z_0 - (I + J_i^-)z_0$$
$$= z_0(J_i^+ - J_i^-). \quad (20)$$

The theory assumes that a linear judgment function maps this affective response onto an overt response. Under this assumption, the overt response, R_i, is

$$R_i = k(J_i^+ - J_i^-), \quad (21)$$

where $k > 0$ is a proportionality constant. Henceforth we assume $k = 1$ for simplicity and describe r_i and R_i interchangeably. Hence, the theory predicts that the affective response to a risky alternative is proportional to the difference between the affectively charged inputs that correspond to attended events. Such differences provide one basis for calling the present formulation affective balance theory.

We now turn to the temporal structure of the events that are assumed to influence gated dipole dynamics under conditions of risky choice. Two temporal sequences of event comparisons are considered: "between-dimension, within-alternative" comparisons and "within-dimension, between-alternative" comparisons (Payne, 1980). Conditions under which both sequences yield the same preference order are described.

In a "between-dimension, within-alternative" comparison, both events within one experimentally grouped pair are first processed before both events in the other experimentally grouped pair are processed. In these experiments, each event in a pair has a manifest affective value of opposite sign; for example, one event represents a possible gain and the other event represents a possible loss. In a "within-dimension, between alternative" comparison, one event within each experimentally grouped pair is processed before it is compared with the event of corresponding manifest value in the other pair. Then the other events in each pair are compared. In the "between-dimension, within-alternative" comparison the decision maker initially samples or attends to one of the alternatives. This sampling causes a net response to that alternative. (In Appendix A we show that choice is independent of which alternative is sampled first.) It is assumed that attention is focused on the first alternative for a sufficient amount of time for the transmitter levels in each channel to habituate, or adapt, to the positive and negative inputs from this alternative. It is assumed that attention next shifts to the second alternative. This attentional shift causes a second net response that is also the result of competition between the gated inputs of the alternative. Because the transmitter levels are slowly varying in time, the inputs of the second alternative are gated by levels that are a function of the inputs of the first alternative. Thus the first alternative establishes an *affective context* in which the second alternative is evaluated.

To see how such an affective context influences judgments, we compute the on-activity r_1 due to the first alternative and the on-activity r_2 due to the second alternative. To accomplish this, denote by z_1^+ the habituated transmitter level in response to J_1^+ and by z_1^- the habituated transmitter level in response to J_1^-. It follows, as in Equations 8 and 9, that

$$z_1^+ = \frac{AB}{A + I + J_1^+} \tag{22}$$

and

$$z_1^- = \frac{AB}{A + I + J_1^-} \tag{23}$$

Thus the on-response r_1 to the inputs J_1^+ and J_1^- is

$$r_1 = (I + J_1^+)z_1^+ - (I + J_1^-)z_1^-. \tag{24}$$

As in Equation 12, Equation 24 may be rewritten in the form

$$r_1 = AC(J_1^+ - J_1^-), \tag{25}$$

where

$$C = \frac{AB}{(A + I + J_1^+)(A + I + J_1^-)} \tag{26}$$

In order to compute r_2, we let transmitter levels z_1^+ and z_1^- gate the inputs J_2^+ and J_2^- of the second alternative. Thus

$$r_2 = (I + J_2^+)z_1^+ - (I + J_2^-)z_1^-. \tag{27}$$

By Equations 22, 23, and 27,

$$r_2 = C(D - E + F), \tag{28}$$

where

$$D = (A + I)(J_2^+ - J_2^-), \tag{29}$$

$$E = I(J_1^+ - J_1^-), \tag{30}$$

and

$$F = J_1^- J_2^+ - J_1^+ J_2^-. \tag{31}$$

We assume that the decision maker senses the difference between the affective responses r_1 and r_2 and prefers the larger one. In this very limited sense, we assume that the decision maker attempts to "maximize" subjective value. Thus, letting $\Delta = r_1 - r_2$, the first alternative is chosen if $\Delta > 0$ and the second alternative is chosen if $\Delta < 0$. Subtracting Equation 28 from Equation 25 yields

$$\Delta = G[(J_1^+ - J_1^-) - (J_2^+ - J_2^-)] + C(J_1^+ J_2^- - J_1^- J_2^+), \tag{32}$$

where

$$G = (A + I)C. \tag{33}$$

The preceding analysis considers the situation in which individuals engage in "between-dimension, within-alternative" processing, where the first alternative establishes the affective context in which the inputs of the second alternative are evaluated. When decision makers utilize a "within-dimension, between-alternative" processing strategy, the sampling sequence first focuses attention on one of the inputs of the first alternative and then shifts attention to the input of the second alternative with the same affective sign. For example, attention may be first focused on the positive input of the first alternative and then shifted to the positive input of the second alternative. Subsequently, attention is focused on the other input of the first alternative and then shifted to the corresponding input of the second alternative.

In these circumstances, the theory assumes that two contexts are established: a positive context in which the positive input of the second alternative is evaluated, and a negative context in which the negative input of the second alternative is evaluated. This computation is instantiated as follows. Denote the positive responses as r_1^+ and r_2^+ and their difference as $\Delta^+ = r_1^+ - r_2^+$. Because r_1^+ is the first term of Equation 24 and r_2^+ is the first term of Equation 27, this difference is simply

$$\Delta^+ = H(J_1^+ - J_2^+) \tag{34}$$

where

$$H = \frac{AB}{A + I + J_1^+}. \tag{35}$$

Denote the negative responses as r_1^- and r_2^- and their difference as $\Delta^- = r_1^- - r_2^-$. Because r_1^- is the second term of Equation 24 and r_2^- is the second term of Equation 27, this difference is simply

$$\Delta^- = K(J_1^- - J_2^-)$$

where

$$K = \frac{-AB}{A + I + J_1^-}. \quad (37)$$

Finally, we assume that preference is determined by the difference between Δ^+ and Δ^-. Subtracting Equation 36 from Equation 34 yields

$$\Delta^+ - \Delta^- = H(J_1^+ - J_2^+) - K(J_1^- - J_2^-). \quad (38)$$

By Equations 35, 37, and 38,

$$\Delta^+ - \Delta^- = G[(J_1^+ - J_1^-) - (J_2^+ - J_2^-)] + C(J_1^+J_2^- - J_1^-J_2^+). \quad (39)$$

Because Equations 39 and 32 are identical, it follows that preference is independent of the decision maker's processing strategy.

We believe that the mechanism that stores the value Δ^+ for later comparison with Δ^- in Equation 38 may not be the same as the mechanism that compares r_1 with r_2 in Equation 32. In particular, Δ^+ may be stored and later read out by a perceptual mechanism, much as a subject can discriminate and remember the intensity of an affectively charged event, such as a shock. In contrast, comparison of r_1 and r_2 can be accomplished directly within a gated dipole.

9. Risk Aversion and Preference Reversals

Evaluation of Equation 32 provides some important insights. Consistent with Equation 21, let us call the difference of the inputs of the ith alternative, $J_i^+ - J_i^-$, the *value* of the ith alternative. From Equation 32, we see that choice is not simply a function of the difference of the values of the alternatives. Instead, choice also depends on the cross-product, $J_1^+J_2^- - J_1^-J_2^+$, and hence on the ratios of the inputs. To illustrate this property, assume that the values of the alternatives are equal or that $J_1^+ - J_1^- = J_2^+ - J_2^-$. Under this assumption,

$$\Delta = C(J_1^+J_2^- - J_1^-J_2^+). \quad (40)$$

From this expression, it is clear that $\Delta \geq 0$ if and only if $J_1^+/J_1^- \geq J_2^+/J_2^-$. By extension, if the difference between the values of the alternatives is small, then choice is primarily determined by the cross-product, or ratio, terms of Equation 32.

The implications of this property represent a break with previous models, because it is possible to choose the second alternative ($\Delta < 0$) even if the first alternative has higher value ($J_1^+ - J_1^- > J_2^+ - J_2^-$). Another implication is that choice between equally valued alternatives should not be random, but rather should be determined by the ratios of the inputs. In Appendix B, we prove the following consequences of this property.

Risk Aversion and Risk Attraction

The riskier of two equally valued alternatives is chosen when both alternatives are viewed as negative, whereas the less risky alternative is chosen when both are viewed as positive.

Preference Reversal

Preference reversals occasionally occur but only when ratios of the inputs are not congruent with the differences of the inputs. That is, preference reversals do not occur when the sign of the differences is consistent with the sign of the ratios. Preference reversals occur in favor of the riskier alternative only when both alternatives are viewed as negative. Preference reversals occur in favor of the conservative alternative only when both alternatives are viewed as positive.

Together, these results illustrate how the theory predicts that individuals will tend to take risks when a situation is unfavorable but tend to be conservative when a situation is favorable. This result is a generalization of the Kahneman and Tversky reflection effect (Section 1). The reflection effect involves choices between risky and riskless alternatives. Our result applies not only to that case (Section 12) but also to choices between pairs of risky alternatives. This generalization is possible because the dynamics of affective choice described herein reveal properties that are not captured by the formal axioms of prospect theory.

The following constraint limits the generality of our analysis. The theory assumes that responses to risky alternatives are due only to the manner in which perceptually driven cognitive processes elicit different affective reactions. Clearly this assumption is not always justified. Certain individuals, such as professional decision analysts, may base their judgments and choices on overt mathematical computations. In these cases, the preceding predictions do not hold because responses are then a function only of the results of the computations. However, the failures of expected value theory and expected utility theory demonstrate that a computationally based approach to decision making under risk is often the exception rather than the rule.

10. Preference Reversal Experiments Demonstrating Interaction Between Hedonic Sign and Risk Aversion

In the following section, we summarize a key result from a pair of experiments that have tested the risk-aversion and risk-attraction properties of preference reversals postulated by affective balance theory. For a detailed discussion of experimental procedures and related results, see Gutowski (1984) and Gutowski and Chechile (1986). The general format for these experiments was a computerized card tournament during which subjects played a modified version of the game called "red-dog" (Epstein, 1977) and gained or lost points that were later exchanged for money. Each alternative consisted of a red-dog hand and a specification of the number of points that could be won or lost. In order to motivate careful judgments, the Marschak bidding technique was employed (Becker, DeGroot, & Marschak, 1964). Both judgment and choice experiments were carried out to provide a stringent criterion for the occurrence of a preference reversal. Judgment and choice trials were interlaced in order to avoid confounding effects due to motivational or attentional shifts.

On each trial of the judgment experiment, a subject first judged the value of the current alternative and then estimated the chance of winning. Subsequently, a random number was generated and if the number was less than the judged value, the hand was played and points were gained or lost depending on whether the subject won the hand. If the random number was less than the judged value, the subject earned that (random) number of points. This procedure was used because it provided

Table 1
Preference Reversal Rates in Favor of the Riskier and Less Risky Alternative as a Function of the Hedonic Sign of the Alternatives

Judged inequality	Hedonic sign of the alternatives: Unfavorable	Favorable
$V_1 < V_2$	0.100	0.679
$V_1 > V_2$	0.528	0.000

Note. V_1 is the judged value of the less risky alternative, and V_2 is the judged value of the riskier alternative.

explicit motivation to give judgments that were exactly equal to the perceived value of an alternative.

On each trial of the choice experiment, subjects were presented with pairs of risky alternatives and asked to indicate which they preferred. On each trial, a subject first indicated the alternative he or she preferred to play and then estimated the chances of winning the chosen hand. After this estimate was recorded, the hand was played and feedback was provided as to the number of points gained or lost. The overall choice set was composed of two groups in which either expected value and the probability of winning were held constant or expected value was held constant but the probability of winning was varied. Many of these pairs of alternatives were chosen to fulfill a property called natural risk ordering. The notation $A_i = (a_i^+, P_i, a_i^-)$ designates an ith alternative in which an amount a_i^+ can be gained with probability P_i and an amount a_i^- can be lost with probability $1 - P_i$. A pair of risky alternatives, $A_1 = (a_1^+, P_1, a_1^-)$ and $A_2 = (a_1^+, P_2, a_2^-)$, is termed *naturally risk ordered* with A_1 riskier than A_2 if and only if $P_1 \leq P_2$ and $a_1^- \leq a_2^-$ and one of these inequalities is strict. For example, the pair $A_1 = (30, 1/3, -10)$ and $A_2 = (50, 1/3, -20)$ is naturally risk ordered because $P_1 = P_2$ and $a_1^- > a_2^-$. In contrast, the pair $A_1 = (50, 1/6, -10)$ and $A_2 = (10, 2/3, -20)$ is not naturally risk ordered because $P_1 < P_2$ but $a_1^- > a_2^-$. For such pairs of alternatives, the risk-aversion and risk-attraction properties of preference reversals proposed by the model can be evaluated even when psychometric judgments of risk are not available.

Differences in the values of a particular pair of alternatives were assumed only if one of the alternatives was judged to be greater than or equal to the other alternative in each replication and if that inequality was strict in at least two of the replications. Preference, as indicated by the choice measure, was assumed only if one of the alternatives was always selected over the other alternative. Otherwise, indifference was assumed in regard to preference or differences in subjective value. Similar criteria were imposed to determine whether a particular alternative was hedonically positive, neutral, or negative. The assumption of hedonic neutrality (i.e., subjective value equal to 0) was rejected only if, for each replication, an alternative was judged to be greater than 0 or judged to be less than 0.

The primary results from these experiments are shown in Table 1, which gives the preference reversal rates as a function of hedonic sign and the judged relation between the values of the riskier and less risky alternative. These data demonstrate that preference reversals in favor of both the riskier and less risky alternatives are strongly related to the hedonic sign of the alternatives. For unfavorable alternatives, the preference reversal rate in favor of the riskier alternative was 0.528, whereas the preference reversal rate in favor of the less risky alternative was only 0.100. This difference is statistically reliable, $\chi^2(1) = 4.80$, $p < .05$. For favorable alternatives, the preference reversal rate in favor of the riskier alternative was 0.000, whereas the reversal rate in favor of the less risky alternative was 0.679. This difference is also statistically reliable, $\chi^2(1) = 7.18, p < .01$. Moreover, the preference reversal rates in favor of the riskier alternative are greater for unfavorable than for favorable alternatives, $\chi^2(1) = 14.56, p < .01$. In contrast, the preference reversal rates in favor of the less risky alternative are greater for favorable than for unfavorable alternatives, $\chi^2(1) = 11.63, p < .01$.

In summary, these results show that the preference reversal phenomenon is a highly structured effect reflecting a willingness to accept risk under unfavorable circumstances and a corresponding unwillingness to accept risk under favorable circumstances. This data pattern is implied by Equation 32. It provides strong support for affective balance theory, because it is a natural consequence of the theory's mechanisms of opponent processing. Such data also challenge other theories of decision making under risk. Subsequent sections describe other important data properties that can naturally be derived from the theory.

11. Gambler's Fallacy: Cognitive-Emotional Dissociation or Partial Reward Effect

The gambler's fallacy is a frequently discussed phenomenon (Cohen, 1981; Diaconis & Freedman, 1981; Skyrms, 1981) that vividly illustrates how decision making under risk may be sensitive to contextual effects generated by a sequence of positive and/or negative outcomes. In general, the gambler's fallacy involves a shift in the amount of risk that a decision maker will accept after a homogeneous sequence of losses (or gains) relative to the amount of risk that will be accepted after a mixed sequence that involves both losses and gains. For example, a roulette player betting on colors might bet $10 on black after four consecutive losses, whereas he or she might bet only $5 after a sequence of two wins and two losses.

Explanations of the gambler's fallacy often assume that individuals do not appreciate the inherent variability of random processes and, thus, inappropriately shift their subjective probabilities after a homogeneous sequence of outcomes (Tversky & Kahneman, 1974). Affective balance theory offers two (experimentally testable) alternative explanations of this phenomenon. The first account is based on a dissociation between long-lasting emotional habituation and cognitive-emotional learning. The second account is based on the antagonistic rebound effects due to disconfirmation of cognitive expectancies. The two accounts are not mutually exclusive in the sense that each may apply to different classes of individuals.

The first account implicates slow recovery from transmitter habituation as a primary mechanism, and a dissociation of transmitter habituation from cognitive-emotional, or conditioned reinforcer, conditioning as a secondary mechanism. This analysis assumes that each win or loss can significantly deplete the transmitter gate of the corresponding on-channel or off-channel, respectively, and that the transmitters recover at a slow rate from these losses. A sequence of wins or losses may consequently cause a cumulative habituation that acts as an affective baseline for subsequent decisions. When such a sequence is homogeneous (e.g., involves all negative outcomes), then the transmitter gate in the off-channel can habituate significantly more than that in the on-channel. When a sequence is mixed (e.g., involves equal numbers of positive and negative outcomes), then the transmitter gates in both the on-channel and the off-channel can habituate significantly. In particular, denote by M a mixed sequence and by N a sequence of losses. A natural outcome of cumulative habituation is that the depletion of the transmitter in the negative channel can be greater after a sequence of losses than after a mixed sequence. Similarly, the depletion of transmitter in the positive channel can be less after a sequence of losses than after a mixed sequence. Formally, we may express these relationships as

$$z^+(M) < z^+(N) \quad \text{and} \quad z^-(M) > z^-(N). \tag{41}$$

Suppose, moreover, that these sustained habituative effects can occur without altering the conditioned reinforcer properties of the bets. In other words, the decision maker codes the event in terms of odds or other computations but does not learn from previous emotional consequences of these computations. Then the inputs J^+ and J^- that a bet generates at the gated dipole do not change over time, but the sensitivity of the dipole to these emotionally changed inputs does change through time.

Under these conditions, the gambler's fallacy occurs if the decision maker continues to bet based on an affective "intuition." This is true because the on-response $r(N)$ to the same inputs (J^+, J^-) after a sequence of losses N is greater than the on-response $r(M)$ to (J^+, J^-) after a mixed sequence M of wins and losses. Consequently, a shift toward larger bets can occur after more losses are experienced under these conditions.

Formally, the gambler's fallacy occurs when

$$r(N) > r(M), \tag{42}$$

where $r(P)$ is the on-response to sequence P. Given Equation 41 and the hypothesis of no conditioned reinforcer learning, Equation 42 follows from the formulas

$$r(N) = (I + J^+)z^+(N) - (I + J^-)z^-(N) \tag{43}$$

and

$$r(M) = (I + J^+)z^+(M) - (I + J^-)z^-(M). \tag{44}$$

In summary, a dissociation between emotional adaptation and cognitive-emotional conditioning can lead to the gambler's fallacy.

We now contrast this result with two examples that illustrate how conditioned reinforcer learning at cognitive-emotional synapses can counteract the tendency to succumb to the gambler's fallacy. First, consider the situation in which the gated dipole quickly overcomes its transmitter habituation but a cumulative record of past wins and losses is encoded within the conditioned reinforcer long-term memory traces (Figure 4). In particular, assume that a series of wins tends to augment J^+, whereas a series of losses tends to augment J^-. Then after a mixed sequence M, these long-term memory increments may (approximately) balance out, so that

$$r(M) \cong \frac{AB(J^+ - J^-)}{A + I} \tag{45}$$

whereas after a sequence of losses N,

$$r(N) \cong \frac{AB[J^+ - (J^- + g(N)]}{A + I}, \tag{46}$$

where $g(N)$ codes the extra conditioned reinforcer negativity due to N. In this situation, the gambler's fallacy fails to hold.

A comparison of these two cases suggests that a persistent emotional desensitization in the absence of cognitive-emotional learning leads to the gambler's fallacy, whereas cognitive-emotional learning accompanied by rapid emotional recovery does not. It remains to consider the case in which transmitter habituation does persist but conditioned reinforcer learning also rescales the inputs (J^+, J^-). In the simplest realization of this interaction,

$$r(M) = \frac{AB(J^+ - J^-)}{A + I + g(M)}, \tag{47}$$

whereas

$$r(N) = \frac{AB(I + J^+)}{A + I} - \frac{AB(I + J^- + C(N)g(N)}{A + I + g(N)}. \tag{48}$$

Term $-C(N)g(N)[A + I + g(N)]^{-1}$ in Equation 48 expresses the negative conditioned reinforcer learning that can compensate for the transmitter habituation terms $g(M)$ and $g(N)$. A large value of $C(N)$ can prevent the gambler's fallacy from occurring.

What factors can control $C(N)$, apart from a simple gain control bias that favors learning over habituation? The answer to this question leads to a second explanation of the gambler's fallacy. A factor of special interest concerns the role of cognitive expectancies in modulating affective reactions. In addition to the direct effects of a win or a loss on habituation of the corresponding transmitter and conditioned reinforcer learning, disconfirmation of an expectancy can cause an antagonistic rebound (Grossberg, 1980, 1987).

Suppose, for definiteness, that the decision maker expects a win. Then a win causes a direct activation of the on-channel. On the other hand, a loss can activate the off-channel both directly and indirectly. The direct effect is due to the loss itself. The indirect effect is due to the off-rebound caused by disconfirmation of the expected win. Consequently, unexpected losses can be more punishing than expected losses and can cause more negative conditioned reinforcer learning. In Equation 48, such an effect can cause a large value of $C(N)$ and can thereby prevent the gambler's fallacy from occurring. This analysis assumes that the decision maker acts with an expectation of winning on every trial. Such a pattern of expectations works against persistent gambling after a long run of losses.

Expectations can, however, shift as a function of the temporal patterning of past wins and losses. Such a hypothesis has been used to suggest an explanation of the partial reinforcement acquisition effect (Grossberg 1975, 1982b). In this analysis, expectations shift with the pattern of wins and losses, and both direct and indirect (antagonistic rebound) effects can amplify the affective charge of an unexpected event. In the classical partial reinforcement paradigm, animals eventually run faster to partially reinforced (mixed sequence) goals than they do to continuously reinforced (positive sequence) goals. If we replace the positive sequence by a negative sequence, then we arrive at an analog of the gambler's fallacy. In this analog, continuous losses (the N sequence) may generate less negative affect than a combination of wins and losses (the M sequence). Such an effect is due to several interacting factors. An enhanced negative affect, or frustration, can be generated by losses on mixed trials, because mixed trials tend to support the maintenance of an expectation to win. This enhanced negative affect may be conditioned, for example, to situational cues, thereby creating a growing baseline of enhanced negativity in the M condition. As in the animal model of partial reinforcement, such a gambler's fallacy develops most effectively if the reinforcement probabilities are initially shaped to generate expectations capable of maintaining such performance.

In summary, the gambler's fallacy may follow either from an abnormal dissociation of cognitive-emotional learning from a persistent emotional habituation or from a reinforcement schedule that sets up the types of correlations between expectancy-modulated antagonistic rebounds and cognitive-emotional learning that support the partial reinforcement acquisition effect.

12. Choice Between Riskless and Risky Alternatives

We next describe some event groupings that may underlie the choice between a riskless alternative and a risky alternative. Suppose, for example, that the riskless alternative is

A. A sure win of $500,

and that the risky alternative is

B. A 3/4 probability of winning $800 and a 1/4 probability of losing $400.

In itself, the riskless alternative does not contain a source of negative affect. On the other hand, a choice of Alternative B implies the renunciation of Alternative A and with it the possibility of a sure win. Embedding a sure win into a context of other possibilities may thus create a source of negative affect.

Expressed in another way, a riskless alternative does not, in itself, involve a comparison between pairs of outcomes in the way that a risky alternative does. The subject must create a grouping of outcomes that includes the riskless alternative in order to generate a preference order. Such a grouping may, or may not, be based entirely on outcomes that are presented explicitly by the experimentalist or more generally by the external environment. In particular, if the subject generates a preference order based on pair-wise comparisons, then a second outcome must be found for comparison with the riskless alternative. One possible outcome, which is not explicitly presented as part of Alternatives A and B, is the denial, or disconfirmation, of the riskless alternative if the risky alternative is accepted.

In the subsequent paragraphs, we consider the preference order generated by two possible event groupings. In the first grouping, the disconfirmation of the riskless alternative is included. In the second grouping, it is not. Consider a situation wherein a decision maker initially samples or attends to the riskless alternative. Heuristically, we may think of the riskless alternative as forming a reference point or adaptation level against which the risky alternative is compared or evaluated. It is assumed that attention is focused on the riskless alternative for a sufficient amount of time for the transmitter levels in the perturbed channel to adapt to the input from the riskless alternative. As attention switches to the risky alternative, two effects occur that combine to determine the net response to the risky alternative. The first effect is an antagonistic rebound within the affective gated dipole. This rebound may occur when the subject switches from the input due to the riskless alternative to the input due to the risky alternative. Such a rebound encodes the affective residue due to a disconfirmation of the riskless alternative in order to evaluate the risky alternative. The second effect is a direct affective response to the inputs from the risky alternative. The net response to the risky alternative is then the sum of these two effects.

Let r_1 be the response to the riskless alternative, $r_1^\dagger$ be the antagonistic rebound, and r_2 be the response to the input of the risky alternative. We may express the net response to a riskless alternative that offers a sure gain as

$$r_1 = AKJ_1^+ \tag{49}$$

where

$$K = \frac{AB}{(A+I)(A+I+J_1^+)}. \tag{50}$$

Equation 49 is just Equation 25 with $J_1^- = 0$. The antagonistic rebound that occurs in response to attention shift to the risky alternative is (approximately)

$$r_1^\dagger = -IKJ_1^+. \tag{51}$$

After transmitter habituation occurs in response to the riskless alternative, the direct response to the inputs J_2^+ and J_2^- of the risky alternative is

$$r_2 = K[(A+I)(J_2^+ - J_2^-) - J_1^+J_2^- - IJ_1^+]. \tag{52}$$

Due to the antagonistic rebound (Equation 51), the net affect due to disconfirmation of the riskless alternative and instatement of the risky alternative is

$$r_1^\dagger + r_2 = K[(A+I)(J_2^+ - J_2^-) - J_1^+J_2^- - 2IJ_1^+]. \tag{53}$$

As in Section 8, we assume that the decision maker senses or computes the difference $\Delta^+ = r_1 - (r_1^\dagger + r_2)$ between these two net responses. By Equations 49 through 53,

$$\Delta^+ = L[J_1^+ + (J_2^+ - J_2^-)] + K(J_1^+J_2^- + IJ_1^+) \tag{54}$$

where

$$L = (A+I)K. \tag{55}$$

A similar analysis may be conducted for a situation where the riskless alternative involves a loss. Under these circumstances, the difference between the two net responses is

$$\Delta^- = L[(J_2^- + J_2^+) - J_1^-] - K(J_1^-J_2^+ + IJ_1^-). \tag{56}$$

One special case of particular importance involves a choice between a riskless alternative and a risky alternative offering only outcomes that are null or of the same sign as the riskless alternative. Assume that only positive or null outcomes are possible; that is,

$$J_2^+ \geq J_2^- = 0. \tag{57}$$

In this case, the response to the riskless alternative and the antagonistic rebound are as expressed in Equations 49 and 51, respectively. The response to the input from the risky alternative is

$$r_2 = K((A + I)J_2^+ - IJ_1^+). \tag{58}$$

Hence, the net response to the risky alternative is

$$r_1^* + r_2 = K[(A + I)J_2^+ - 2IJ_1^+] \tag{59}$$

so that the difference between the two net responses is

$$\Delta_0^+ = L[J_1^+ - J_2^+] + KIJ_1^+. \tag{60}$$

If only negative or null outcomes are possible, then this difference is

$$\Delta_0^- = L[J_2^- - J_1^-] - KIJ_1^-. \tag{61}$$

The implications of this analysis are applicable to two important phenomena that have been discovered during the development of prospect theory. Consider the following problems (from Tversky & Kahneman, 1981):

Which of the following alternatives do you prefer?

Problem 1
A. A sure win of $3000.
B. An 80% chance to win $4000.

Problem 2
C. A sure loss of $3000.
D. An 80% chance to lose $4000.

When presented with these problems, the majority of individuals prefer A to B and D to C. That is, individuals tend to be risk averse when gains are involved but risk taking when losses are involved. The previous computation accounts for this effect as the net effect of two types of factors: affective antagonistic rebounds that occur when sure events are contextually disconfirmed, and affective reactions to risky events based on prior affective adaptations. Suppose, for example, in Problem 1, that attention is shifted from the riskless alternative to the risky alternative. The rebound is negative, thereby decreasing the net response to the risky alternative. Consequently, the riskless alternative will be chosen unless r_2 is considerably larger than r_1; that is, the risk-averse alternative is preferred where gains are involved. In the second problem, the rebound due to an attention shift from the riskless alternative is positive, which increases the net response to the risky alternative. As a result, the risky alternative will be chosen unless r_2 is considerably smaller than r_1; that is, the risk-taking alternative is preferred where losses are involved.

We consider it possible that some subjects may affectively evaluate a risky alternative by switching rapidly between its two possible interpretations. For example, an 80% chance to win $4000 may be represented as an input with conditioned reinforcer value appropriate to $4000 rapidly alternating with an offset of that input (20% chance of winning nothing). Where such rapid switching occurs, the subject creates an implicit conflict situation by generating off-rebounds to the positively valenced response. It is worth considering whether some subjects encode the 80%–20% contingency by just partially switching off the positively valenced signal, and thereby weakening the off-rebounds that compete with the positively valenced reaction before generating a net dipole output.

In the second sequence of comparisons that we will consider, the subject first adapts to the riskless alternative and then switches to the risky alternative without processing the disconfirmation that rejection of the riskless alternative could imply. All of the above computations are the same, except there is no longer an antagonistic rebound r_1^*. Then Equation 54 is replaced by

$$\Delta^+ = L[J_1^+ - (J_2^+ - J_2^-)] + J_1^+J_2^- \tag{62}$$

and Equation 60 is replaced by

$$\Delta_0^+ = L[J_1^+ - J_2^+]. \tag{63}$$

When Equation 63 is used to explain why Alternative A is preferred to Alternative B, the psychophysical properties of the Kahneman and Tversky axioms (Section 7) bear the full burden of the explanation.

A similar type of contextual effect occurs if a positive riskless alternative is compared sequentially with the positive and negative events of a risky alternative. Then each of the comparisons may generate antagonistic rebounds due to the fact that the riskless alternative defines an adaptation level for evaluating both risky events.

In summary, this section illustrates how the additional affective values that are generated by contextually induced comparisons may explain certain properties of decision making under risk without invoking all of the Kahneman and Tversky axioms.

13. Framing Effect

Next, consider the following problems (from Tversky & Kahneman, 1981) that illustrate the framing effect.

Problem 3
Imagine that the United States is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences are as follows:

If Program A is adopted, 200 people will be saved.

If Program B is adopted, there is a 1/3 probability that 600 people will be saved, and a 2/3 probability that no people will be saved.

Problem 4
Given the same cover story as in Problem 3, a different formulation of the net effects of the alternative programs is:

If Program C is adopted, 400 people will die.

If Program D is adopted, there is a 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.

When presented with these problems the majority of individuals prefer A to B and D to C. That is, individuals tend to be risk averse when only positive consequences are explicitly mentioned but risk taking when only negative consequences are presented. The explanation for this framing effect is dynamically similar to that offered for Problems 1 and 2. At least two different types of strategies exist that are consistent with the framing effect. In both strategies, subjects are assumed to respond to riskless alternatives by discovering other events with which they can be paired. The strategies differ only in terms of which event pairs are considered. For example, in Problem 3, the event that 600 people will be killed may be treated as a riskless alternative against which Programs A and B are evaluated. Within this context, the statement "If Program A is adopted, 200 people will be saved" creates a natural event pair, or a contextually induced alternative. In particular, disconfirmation of the riskless loss of 600 lives by the riskless gain of 200 lives can trigger a large antagonistic rebound. The contextual pressure to form such an event pair is substantial in this situation because the statement "200 people will be saved" is meaningless without a prior statement concerning their expected doom.

The evaluation of Program B may, in principle, be made with respect to the sure event that 600 people will be killed, or the sure event that 200 people will be saved, or both. To fix ideas, suppose that the subject creates pair-wise comparisons where none explicitly exist in the stimuli but otherwise considers the events sequentially. Then, after the comparison of the sure death of 600 with Program A generates a large positive rebound, comparison of Program B with Program A generates an additional bias toward Program A, because this comparison involves a positive riskless alternative and a risky alternative (Section 12).

In subjects who use the sure death of 600 as a context for evaluating Alternatives C and D, D is preferred to C for the following reasons. Disconfirmation of the certain death of 600 by instatement of the certain death of 400 people may create a small positive rebound. Comparison of Program D with Program C generates a bias away from Program C, however, because this comparison involves a negative riskless alternative and a risky alternative. Thus, independent of parametric details, it is clear that Program A is much more favored than Program B, whereas there is at best a weak tendency to offset the risk-seeking bias of Program D over Program C. In particular, there exists a range of parameters where the rebound is insufficient to offset the risk-seeking bias.

If a subject does not use the sure event of 600 deaths as a context for further comparisons, then Program A is again preferred to Program B whereas Program D is preferred to Program C because the risk-averse alternative is preferred where gains are involved whereas the risk-taking alternative is preferred where losses are involved. Subjects may, in fact, follow this strategy more readily in comparing Programs C and D than in comparing Programs A and B. Unlike Alternative A, the meaning of Alternative C is self-contained and does require comparison with the riskless death of 600.

14. Concluding Remarks

Affective balance theory provides a real-time dynamic description of some of the covert events underlying risky judgment and choice. The theory proposes dynamical explanations of decision-making phenomena that have previously been interpretable only using formal axioms, such as those of prospect theory. This analysis clarifies how cognitive strategies may generate affective contexts for evaluating riskless and risky alternatives. In so doing, it provides an explanation of why individuals often do not act to maximize subjective value, even in simple situations where cognitive complexity is minimal, despite the fact that the mechanisms that prevent maximization have manifest adaptive functions. These mechanisms are not designed to maximize subjective value. Rather, they are designed to control the emotional processes that regulate reinforcement, incentive motivation, and affectively modulated attention shifts.

Many phenomena still lie outside the scope of the theory as presently formulated, if only because it has used only the most rudimentary ideas from the cognitive-emotional theory of which it is an application. The present results establish a bridge to that general theory and indicate how affective balance theory may be developed on a principled basis as increasingly complex situations are analyzed. In particular, the present results demonstrate that psychophysiological data and theory may now be profitably applied to the domain of decision making under risk.

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Appendix A

Relative Preference Independent of Sampling Order

Consider a choice between two alternatives a_1 and a_2. Let r_{ij} denote the response to the ith alternative when a_j is sampled first. If a_1 is sampled first, then after transmitter habituation takes place,

$$z_1^+ = \frac{AB}{(A+I+J_1^+)}, \quad \text{(A1)}$$

$$z_1^- = \frac{AB}{(A+I+J_1^-)}, \quad \text{(A2)}$$

and

$$r_{11} = AK_1(J_1^+ - J_1^-), \quad \text{(A3)}$$

where

$$K_1 = \frac{AB(J_1^+ - J_1^-)}{(A+I+J_1^-)(A+I+J_1^-)}. \quad \text{(A4)}$$

When attention is then switched to a_2, the response, r_{21}, is based on the habituated values in Equations A1 and A2. Thus

$$r_{21} = K_1[(A+I)(J_2^+ - J_2^-) - I(J_1^+ - J_1^-) + J_2^+J_1^- - J_1^+J_2^-)]. \quad \text{(A5)}$$

The difference $\Delta^{(1)} = r_{21} - r_{11}$ in these responses is

$$\Delta^{(1)} = K_1[(A+I)((J_2^+ - J_2^-) - (J_1^+ - J_1^-)) + J_2^+J_1^- - J_2^-J_1^+]. \quad \text{(A6)}$$

If a_2 is sampled first, then

$$z_2^+ = \frac{AB}{(A+I+J_2^+)}, \quad \text{(A7)}$$

$$z_2^- = \frac{AB}{(A+I+J_2^-)}, \quad \text{(A8)}$$

and

$$r_{22} = AK_2(J_2^+ - J_2^-), \quad \text{(A9)}$$

where

$$K_2 = \frac{AB}{(A+I+J_2^+)(A+I+J_2^-)} \quad \text{(A10)}$$

When attention is then switched to a_1, the response, r_{12}, is

$$r_{12} = K_2[(A+I)(J_1^+ - J_1^- \; - I(J_2^+ - J_2^-) + J_1^+J_2^- - J_1^-J_2^+]. \quad \text{(A11)}$$

The difference $\Delta^{(2)} = r_{22} - r_{12}$ in these responses is

$$\Delta^{(2)} = K_2[(A+I)((J_2^+ - J_2^-) - (J_1^+ - J_1^-)) + J_2^+J_1^- - J_2^-J_1^+]. \quad \text{(A12)}$$

Term $\Delta^{(1)}$ compares the relative reaction to a_2 after a_1. Term $\Delta^{(2)}$ compares the relative reaction to a_2 before a_1. By Equations A6 and A12, these terms differ only in their positive coefficients K_1 and K_2. Thus $\Delta^{(1)}$ is positive (negative) if and only if $\Delta^{(2)}$ is positive (negative). Hence the same relative preference exists independent of sampling order.

Appendix B

Risk Aversion, Risk Attraction, and Preference Reversal

Consider a choice between two equally valued alternatives a_1 and a_2. Let $D = J_1^+ - J_1^- = J_2^+ - J_2^-$. Assume that a_2 is riskier than a_1 so that $J_1^- < J_2^-$. Without loss of generality, assume that a_1 is sampled first. Then by Equation A6,

$$\begin{aligned}\Delta^{(1)} &= K_1[J_2^+J_1^- - J_2^-J_1^+] \\ &= K_1[(J_2^- + D)J_1^- - J_2^-(J_1^- + D)] \\ &= D(J_1^- - J_2^-). \quad \text{(B1)}\end{aligned}$$

Therefore, $\Delta^{(1)} < 0$ if and only if $D > 0$, because $J_1^- - J_2^- < 0$. This result demonstrates that the less risky alternative is chosen ($\Delta^{(1)} < 0$) only if the positive input is greater than the negative input ($D > 0$). That is, the less risky alternative of two equally valued alternatives is chosen only if the alternatives are favorable.

Consider a choice between two alternatives that are not equal in value. Let $J_1^+ - J_1^- = D_1$ and $J_2^+ - J_2^- = D_1 + D_2$. Assume that a_2 is riskier than a_1, so that $J_1^- < J_2^-$. Let $\Delta^{(1)} = r_{21} - r_{11}$, as in Appendix A. A *preference reversal* is said to occur if

$$D_2\Delta^{(1)} < 0; \quad \text{(B2)}$$

that is, if a_2 is preferred over a_1 ($\Delta^{(1)} > 0$) even though a_1 has greater value than a_2 ($D_2 < 0$); or if a_1 is preferred over a_2 ($\Delta^{(1)} < 0$) even though a_2 has greater value than a_1 ($D_2 > 0$).

By Equation A6,

$$\begin{aligned}\Delta^{(1)} &= K_1[(A+I)D_2 + J_1^-(J_2^- + D_1 + D_2) \\ &\quad - J_2^-(J_1^- + D_1)] \\ &= K_1[(A+I+J_1^-)D_2 + (J_1^- - J_2^-)D_1]. \quad \text{(B3)}\end{aligned}$$

Because $A + I + J_1^- > 0$ and $J_1^- - J_2^- < 0$,

$$\Delta^{(1)} > 0 \quad \text{if} \quad D_1 < 0 \quad \text{and} \quad D_2 > 0, \quad \text{(B4)}$$

whereas

$$\Delta^{(1)} < 0 \quad \text{if} \quad D_1 > 0 \quad \text{and} \quad D_2 < 0. \quad \text{(B5)}$$

The difference $\Delta^{(1)}$ may be positive or negative if

$$D_1 D_2 > 0. \quad \text{(B6)}$$

A preference reversal does not occur in either of the cases in Equation B4 or Equation B5. A preference reversal can occur only if Equation B6 holds. By Equations B3 and B6, two possible cases obtain: Either

$$\Delta^{(1)} < 0, D_1 > 0, D_2 > 0 \quad \text{(B7)}$$

or

$$\Delta^{(1)} > 0, D_1 < 0, D_2 < 0. \quad \text{(B8)}$$

In the case of Equation B7, the preference reversal favors the less risky alternative a_1 only if the alternatives are favorable ($D_1 > 0$). In the case of Equation B8, the preference reversal favors the riskier alternative a_2 only if the alternatives are unfavorable ($D_1 < 0$).

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