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DETECTION OF TOOL FAILURE IN END MILLING WITH WAVELET TRANSFORMATIONS AND NEURAL NETWORKS (WT–NN)

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Abstract—Detection of tool failure is very important in automated manufacturing. In this study, tool failure detection was conducted in two steps by using Wavelet Transformations and Neural Networks (WT–NN). In the first step, data were compressed by using wavelet transformations and unnecessary details were eliminated. In the second step, the estimated parameters of the wavelet transformations were classified by using Adaptive Resonance Theory (ART2)-type self-learning neural networks. Wavelet transformations represent transitionary data and complex patterns in a more compact form than time-series methods (frequency and time-domain) by using a family of the most suitable wave forms. Wavelet transformations can also be implemented on parallel processors and require less computations than Fast Fourier Transformation (FFT). The training of ART2-type neural networks is faster than backpropagation-type neural networks and ART2 is capable of updating its experience with the help of an operator while it is monitoring the sensory signals. The proposed approach was tested in over 171 cases and all the presented cases were accurately classified. The proposed system can be easily trained to inspect data during transition and/or any complex cutting conditions. The system will indicate failure instantaneously by creating a new category, thus alerting the operator.

1. INTRODUCTION

Automatic monitoring of tool conditions and detection of tool failure are necessary in automated manufacturing to improve quality and increase productivity. Many different approaches have already been developed to secure a reliable monitoring technique. The value of tool failure detection systems is evaluated according to their reliability, accuracy, speed, simplicity, versatility and ease of programming requirements. In this paper, the feasibility of combining two very promising techniques [Wavelet Transformations and Neural Networks (WT–NN)] is investigated. The WT–NN compresses resultant force by using wavelet transformations and classifies this compact information by using Adaptive Resonance Theory (ART-2)-type self-learning neural networks. The main advantages of the proposed system are its fast algorithm, efficient use of parallel processors and easy implementation to various machining operations simply by on-site training. A multipurpose WT–NN can be used for many other applications, from speech recognition to monitoring drilling operations.

Previously developed tool failure detection systems used either custom developed programs or used a known procedure with some modifications. Altintas *et al.* [1] Altintas and Yellowly [2], Sutherland *et al.* [3] and Richter and Spiewak [4] developed their failure detection algorithms by considering the theoretical force variation characteristics of milling signals. Other researchers developed their methods by using either time-series analysis methods [5–8] or neural networks [9, 10]. To detect tool failure, researchers either forecast the cutting forces periodically by using time-series methods and evaluating the error [5–7] or inspecting the parameters of estimated models [8]. Implementation of the time-series approaches is not very easy since these procedures require preparation of very efficient programs that can perform very fast on-line data

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acquisition and the fitting of high-order models [9]. Properly selected, trained neural networks can estimate tool condition [10, 11] by inspecting the encoded data of a single revolution of the tool. This capability eliminates the on-line data collection requirement and simplifies programming. Neural network-based methods may perform data acquisition in the background and transfer it to memory through the Direct Memory Access (DMA) feature of microcomputers.

During milling operations, unique cutting force or torque signals are created in every tool revolution if the cutting conditions are the same. These signals have a complex pattern [12, 13] that cannot be represented with the addition of a few harmonic functions. The low frequency patterns are generated by the variation of the chip width at each cutting edge of the cutter. The cutting force created by each cutting edge rises and then falls to zero acccording to a complex equation [14] that cannot be represented with a single sine wave. The duration of this pattern is half of the maximum of one tool revolution in slotting. During the other half of the tool rotation, the cutting force is zero for that tooth. Even if all the cutting edges have a certain runout, at most, we expect ascents and descents that are equal to the number of teeth in that cutter. The high frequency patterns are generated by the vibrations, material characteristic variations, and other factors. Technically, the end milling signals can be described as a unique pattern with a large number of time-localized features and many details. To detect tool failure, the unique cutting force pattern has to be isolated and represented by a small number of parameters.

Time-series methods assume that the output of all systems can be represented by totalling the number of harmonic signals. This assumption is valid when the oscillation of a beam is considered. For example, to represent a square wave, FFT suggests addition of a sine wave and several harmonics. For the same signal, time-domain/time-series methods estimate very high order models, which have spectral characteristics similar to FFT.

Recently, wavelet transformations were used to separate the important features of signals from details and to generate a more compact representation. Wavelet transformations have also been used to compress one (sound) and two (digital image) dimensional data [14–16]. Especially when the signal is generated with the addition of complex patterns, more accurate and compact representation is expected from the use of orthonormal basis functions or wavelet transformation [17], since they represent any given signal by translating and scaling a custom designed mother wave [17–23]. For example, wavelet transformations would give a very compact and accurate representation of a square wave better than FFT when the Haar function is used as the mother wave, which is a rectangle. On the other hand, vibrations of linear structures are generated by the addition of harmonic oscillations, and a fixed and narrow resolution is required to estimate the natural frequency of the system. Time-series methods can estimate the natural frequency of the system much more accurately than wavelet transformations.

Typically, wavelet transformations require less computation than FFT. For example, FFT requires $N\log_2 N$ operations for transformation of a set of N numbers. Fast wavelet transformations require N operations, and the number of operations halves when the transformations are repeated [17].

According to the above discussions, wavelet transformations are excellent candidates for preparing a very representative and compact data set for cutting force signals. The compact data set can then be presented to neural networks for classification. The authors used WT–NN previously for detection of tool failure in micro-drilling [24].

Neural networks have been used for a long time for the classification of various signals [25]. The most commonly used neural network in manufacturing related research is backpropagation [26]. Tansel and McLaughlin [10, 11] and Burke [27], however, used ART2-type neural networks for their fast and continuous learning capability. In prevous studies, sensory signals were encoded with a custom-built procedure. The number of inputs to the neural network can be kept to a minimum and computational speed can be increased by using custom-built procedures; however, it requires experience, programming and considerable computer knowledge if parallel processors are used.

In the following section, a theoretical background of wavelet transformations and neural networks will be introduced first. The experimental set-up, the tool failure detection procedure and the results will be presented in subsequent sections.

2. THE THEORETICAL BACKGROUND

The experimental data were encoded by using wavelet transformations, and the encoded data were then presented to the neural networks. A theoretical background of wavelet transformations and ART2-type neural networks will be presented in this section.

2.1. *Wavelet transformations*

Wavelets are obtained by creating a family of functions derived from one single function [20, 21, 23]. This system can be expressed by the following equation:

$$h^{(a,b)}(x) = |a|^{-1/2} h\left(\frac{x-b}{a}\right) , \tag{1}$$

where a and b are the dilation and translation parameters, respectively. In the above equation, $h^{(a,b)}$ represents the family of wavelets obtained from the single h function by dilations and translations. The given data consist of the f function in the given x coordinate.

After wavelet transformation, the original signal can be reconstructed by using the following expression:

$$f = C \int \frac{\mathrm{d}a}{a^2} \int \mathrm{d}b \langle h^{(a,b)}, f \rangle h^{(a,b)} , \tag{2}$$

where f is the original function. $\langle h^{(a,b)}, f \rangle$ are the inner products of the wavelet.

A discrete wavelet transform is used to work with discrete signals [21, 23]. The function, f, can be mapped with sequences under certain conditions and the wavelets can be obtained with the following equation:

$$(Tf)_{mn} = \langle h_{mn}, f \rangle = a_0^{-m/2} \int h(a_0^{-m} x - n b_0) f(x)\, \mathrm{d}x , \tag{3}$$

where $(Tf)_{mn}$ represents the discrete wavelet transformation, and $\langle h_{mn}, f \rangle$ are the wavelet coefficients. The a_0 and b_0 are the fixed dilation and translation steps, respectively. The dilation and translation coefficients have the following relationship with the fixed steps:

$$a = a_0^m , b = n b_0 a_0^m , \tag{4}$$

where m and n are the indices. The original function can be reconstructed from the calculated wavelets with the following expression:

$$f = \frac{2}{A+B} \sum_{m,n} \langle h_{m,n}, f \rangle + R \tag{5}$$

with

$$\|R\| = \langle O\left(\frac{B}{A} - 1\right) \|f\| .$$

It can be proven that based on the above descriptions, the basic function (scaling function), $\Phi(x)$, of a wavelet system can be calculated with the following recursive equation [17, 21, 23]:

$$\Phi(x) = \sum_n c(n)\, \Phi(2x-n) , \tag{6}$$

where $c(n)$ is the wavelet coefficient and n is the index. The primary wavelet, $\Psi(x)$, can be obtained with the following expression:

$$\Psi(x) = \sum_n (-1)^n c(n+1)\Phi(2x-n) , \tag{7}$$

where $c(n + 1)$ is the coefficient. The original function can be reconstructed by the following expression:

$$f(x) = \sum_{n=-\infty}^{\infty} c(n)\Phi_n(t) + \sum_{i=0}^{\infty} \sum_{j=-\infty}^{\infty} d(i,j)\Psi_{i,j}(t) \tag{8}$$

where $c(n) = \int f(t)\phi_n(t)\mathrm{d}t$

and $d(i,j) = \int f(t)\, \Psi_{i,j}(t)\mathrm{d}t$,

where $c(n)$ and $d(i,j)$ are the coefficients of the wavelet transform.

Daubechies [21, 23] proposed a wavelet system based on an orthonormal base. The h functions had the following values for this system:

$$h(0) = \frac{(1 \pm \sqrt{3})}{(4\sqrt{2})}$$

$$h(1) = \frac{(3 \pm \sqrt{3})}{(4\sqrt{2})}$$

$$h(2) = \frac{(3 \pm \sqrt{3})}{(4\sqrt{2})} \tag{9}$$

$$h(3) = \frac{(1 \pm \sqrt{3})}{(4\sqrt{2})} .$$

In this paper, the wavelet coefficients are found based on the above wavelet system.

2.2. *Adaptive resonance theory*

The theory of adaptive resonance networks was first introduced by Carpenter and Grossberg [28] and Grossberg [29, 30]. According to this theory, adaptive resonance occurs when the input to a network and the feedback expectancies match. The ART2-type neural networks were developed by Carpenter and Grossberg [28] to achieve a self-organized stable pattern recognition capability in real time. The ART2-type neural networks compare a given input with previously encountered patterns. If the input is similar to any of the patterns, it will be placed in the same category with similar patterns. On the other hand, if the input is not similar to any of the previously presented patterns, a new category will be assigned to the given input. The sensitivity of the neural network is adjusted with the vigilance value. High vigilance will increase sensitivity, will reduce errors and will generate a large number of categories. The number of assigned categories can be reduced by selecting low vigilance values; however, errors will increase. An optimal vigilance value creates a reasonable number of categories with minimum error.

3. EXPERIMENTAL SET-UP

The experimental apparatus is presented in Fig. 1. A standard Bridgeport Vertical Milling Machine was used for the experimental study. The cutting force signals were measured by using a Kistler 9257A 3-Component dynamometer. The dynamometer was mounted on the table of the milling machine. The workpiece was then mounted on the dynamometer. The cutting force signals were sampled at fixed rotation angles of the spindle (X direction and Y direction force components in the horizontal plane were sampled in turn at every 1° rotation of the spindle) by using a Litton Model 70 encoder as a triggering source. The output voltage signal of the Charge Amplifier was collected separately by both a microcomputer (on-line) and a Tektronix 2221 digital oscilloscope (off-line). The microcomputer had a Metrabyte uCDAS-16G A/D board installed in one expansion slot to sample the data on-line. A Tektronix 2221 digital oscilloscope was used to sample the data for off-line analysis. The resultant force was calculated from the measured cutting force components.

Experimental data were collected using four flute end mills (diameter of 12.7 mm). It was impossible to break one tooth during the experiment to collect the good and broken tool cutting force signals in the same data set with less than a few thousand samples. In this study, exactly the same two end mills were used. One tooth of one of the end mills was ground to eliminate its tool removal capability. The other tool was in excellent condition. The experiments were repeated for the same cutting conditions. The good and broken tool data were carefully assembled later to obtain a data set that starts with the good tool signal and continues with the broken tool signal. The data set represented the necessary data with good and broken tool sections. The cutting conditions and variations of the sum of the squares of the estimation errors of tooth periods are outlined in section 5.

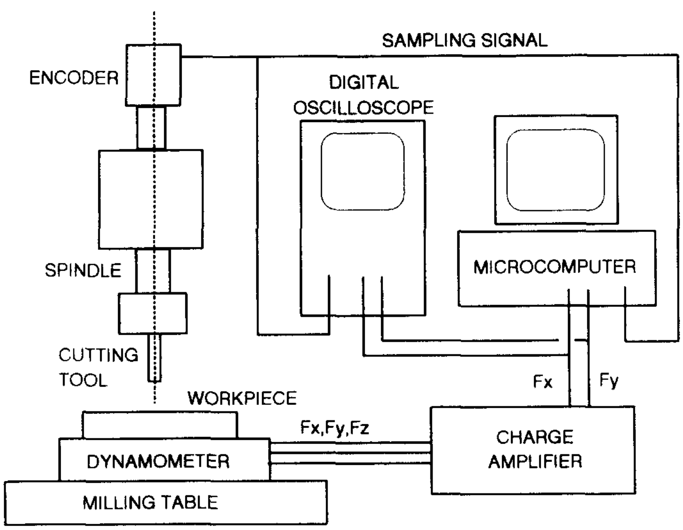

Fig. 1. The experimental set-up.

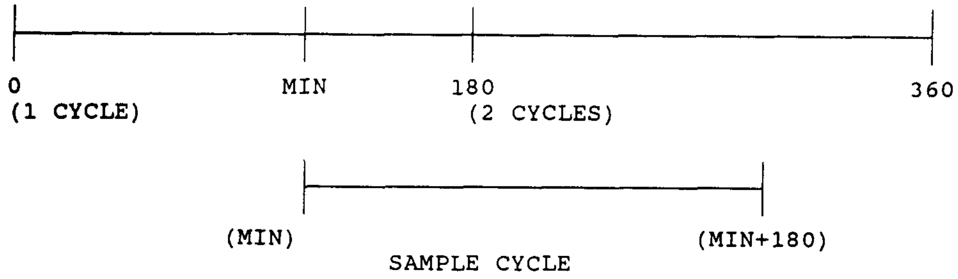

Fig. 2. Selecting the sample cycles.

4. TOOL FAILURE DETECTION

The experimental data were collected by sampling the cutting forces in the horizontal XY plane 180 times in each revolution by using an optic encoder. The cutting forces in the X and Y directions were measured and the resultant forces were calculated. Blocks of 360 data points, representing two complete cycles, were analyzed. The minimum point in the first 180 points (one cycle) was found and then the sample cycle was taken starting from that point (i.e., the next 180 values from the minimum point were used as the sample cycle). A selection of the 180 values is presented in Fig. 2.

For each of the nine cases, it was possible to obtain 19 sample cycles. The wavelet transformation of each of these cycles was found using a Daubechies-type wavelet [please see equation (9)]. The $c(n)$, equation (8), transformation coefficients were obtained after the transformation was repeated four times. These coefficients were used to characterize the sample and they were presented to the ART2-type neural network for classification.

5. RESULTS AND DISCUSSION

In this study, the characteristics of normal and broken tool signals were studied in three steps. First, the pattern of the resultant forces was visually inspected. Secondly the wavelet transformation of the forces was calculated and distinctive patterns were observed. Finally, the wavelet transformations were presented to the neural network and the accuracy of the classifications was evaluated.

The experimental cutting conditions are given in Table 1. Typical data are presented

Table 1. Experimental cutting conditions. Tool diameter is 12.7 mm. Normal tool had four cutting edges. One of the cutting edges of the broken tool did not remove any material

Experiment no.	Axial depth of cut (mm)	Feed rate (mm/min)	Spindle speed (rpm)	Tool condition
1	1.524	101.6	700	Normal
	1.524	101.6	700	Broken
2	1.016	203.2	500	Normal
	1.016	203.2	500	Broken
3	1.524	50.8	500	Normal
	1.524	50.8	500	Broken
4	1.524	101.6	500	Normal
	1.524	101.6	500	Broken
5	1.524	203.6	700	Normal
	1.524	203.6	700	Broken
6	1.524	254	700	Normal
	1.524	254	700	Broken
7	1.016	152.4	500	Normal
	1.016	152.4	500	Broken
8	1.524	152.4	500	Normal
	1.524	152.4	500	Broken
9	1.524	152.4	700	Normal
	1.524	152.4	700	Broken

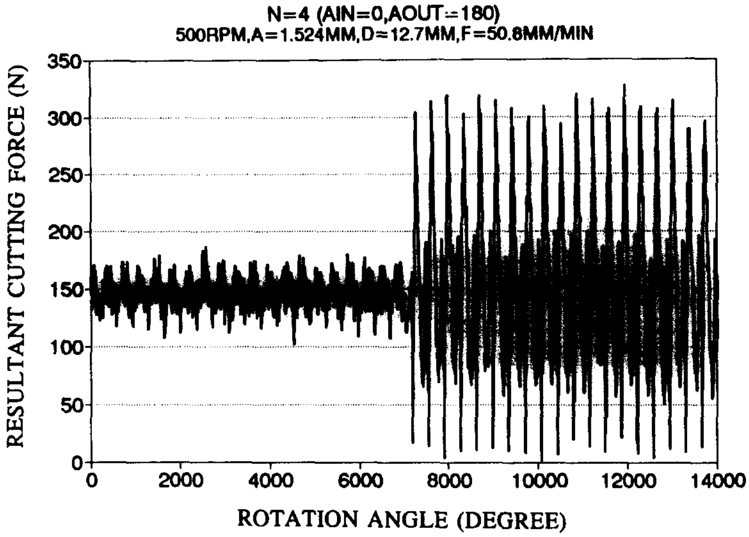

Fig. 3. Variation of the resultant cutting force. The tool is broken in the middle.

in Fig. 3. In the beginning, the variation of the resultant cutting forces was created with runout and vibrations of the system. However, the variations suddenly increased when one of the teeth was broken. The waveform and the variation of the good and broken tool signals were very distinctive.

The $c(n)$ coefficients in equation (8) were calculated for the data in Fig. 3 and are presented in Fig. 4. To prepare the plot, 360 data-point-long data blocks were taken and processed according to the procedure outlined in section 4. On the plot, the 26 $c(n)$ coefficients [equation (8)], and 19 cases were assigned to the X and Y axes, respectively. The values of the parameters are on the Z axis. The values of the estimated parameters were almost the same in the first ten cases while the cutting tool was sharp. When the tool was broken, the values of the 2nd to 11th parameters almost doubled. Figure 4 indicates that by using the wavelet transformation, the data consisting of 180

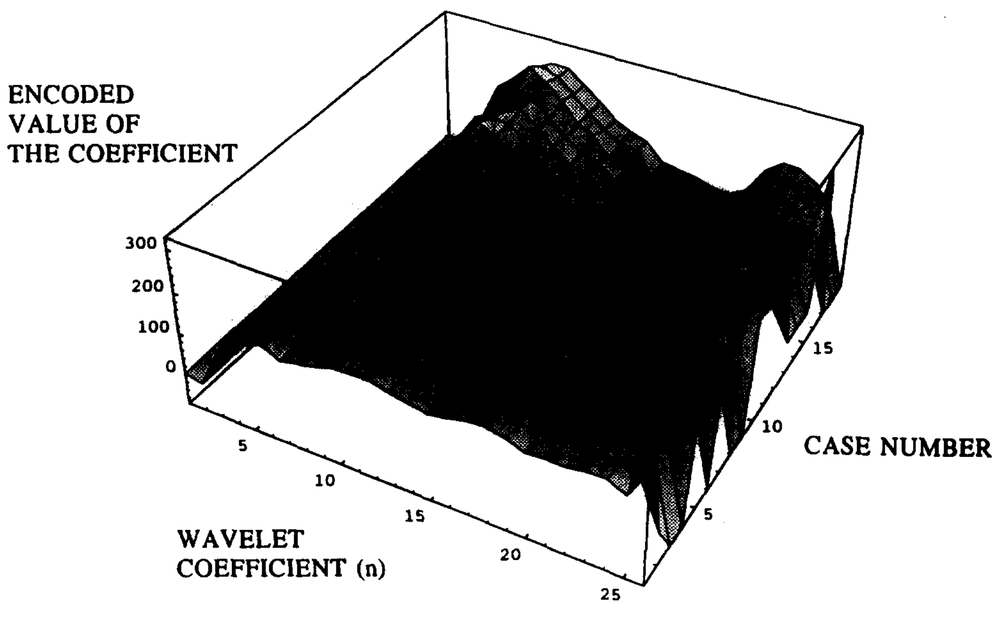

Fig. 4. Variation of the parameters of the wavelet transformation with normal and broken tools. The X and Y axes were assigned to 26 wavelet coefficients and 19 cases. The Z values represent the values of each coefficient.

data points can be compressed into a meaningful set of 26 numbers and these parameters can be used to detect tool failure.

The experiments were repeated for eight more different cutting conditions. At each cutting condition, 19 blocks of data were processed. In each set, either the first nine or ten data blocks were taken with a normal tool. The estimated $c(n)$ parameters of 171 different cases are presented in Fig. 5. Similar to Fig. 4, the X and Y axes were assigned to 26 wavelet coefficients and 171 cases. The Z represents the values of each parameter. The values of the parameters depend on the cutting condition, while the patterns depend on the tool condition. All of the parameters were similar for normal tools. When one of the cutters was broken, either the 3rd to the 12th, or the 20th to the 25th parameters almost doubled. By inspecting Fig. 5, it is possible to identify where the cutting condition changed, or where the tool was broken. This analysis indicates that the $c(n)$ parameters of the wavelet transformations can be used to monitor variations of the cutting, or the tool conditions.

23 parameters (the 2nd to 24th) of the 171 cases were presented to an ART2-type neural network. The neural network identified 18 different cases when the selected vigilance was 0.995 (Table 2). All of the changes in the cutting conditions and the tool failures were identified accurately (ART2 created 18 categories, 1 normal and 1 broken tool category for each of 9 cutting conditions). The study indicates that ART2 can easily detect tool failure and tool conditions when the wavelet transformations were used for encoding.

The ART2 program evaluates each presented case only once. Backpropagation-type neural networks require updating the model many times (iterations) during the training session by using the presented set. Depending on the number of cases in the training set, the number of inputs and outputs, and selected network size, the iterations may reach hundreds of thousands, or even millions. In most of the applications, the training of the ART2 can be completed in a very short time relative to backpropagation-type neural networks.

The advantages of the proposed method are the following:

(a) Implementation of the proposed method is very simple if the proposed WT–NN boards become available (much easier than any other method available, until now [1–8, 10, 11]). At present, separate wavelet transformation and neural net-

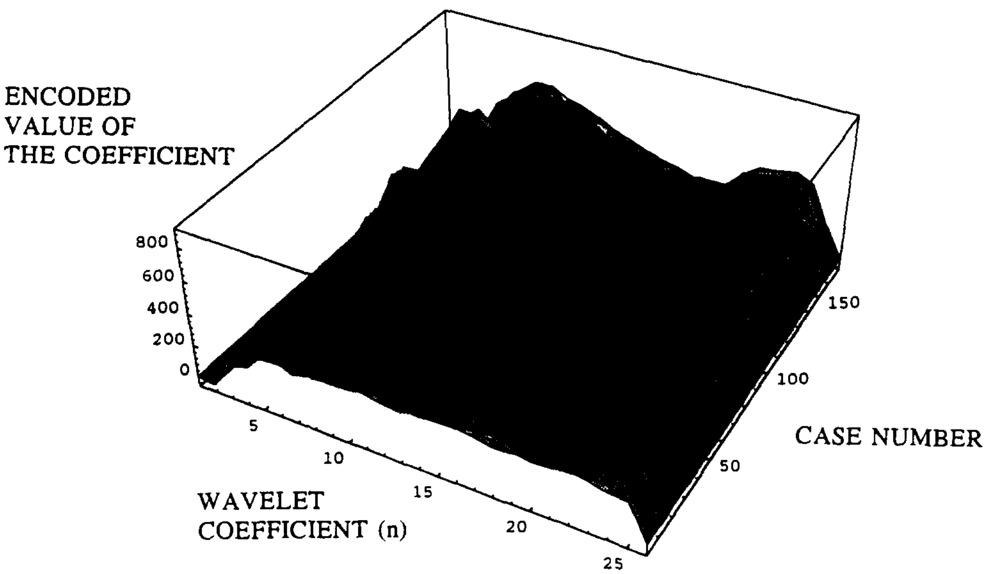

Fig. 5. Variation of the parameters of the wavelet transformation at nine different cutting conditions with normal and broken tools. The X and Y axes were assigned to 26 wavelet coefficients and 171 cases. The Z values represent the values of each coefficient.

Table 2. Classification of the ART2 neural networks with different vigilance values. The cutting conditions for each case are given in Table 1

		Vigilance = 0.995		
Experiment no.	Condition	Presented cases in the same condition	Assigned category by ART2	Wrong classifications
1	Normal	10	1	0
	Broken	9	2	0
2	Normal	10	3	0
	Broken	9	4	0
3	Normal	10	5	0
	Broken	9	6	0
4	Normal	10	7	0
	Broken	9	8	0
5	Normal	10	9	0
	Broken	9	10	0
6	Normal	10	11	0
	Broken	9	12	0
7	Normal	10	13	0
	Broken	9	14	0
8	Normal	10	15	0
	Broken	9	16	0
9	Normal	10	17	0
	Broken	9	18	0

work chips and high-speed hardware are available. In the near future, multi-purpose WT–NN hardware may be developed for many applications, including speech processing, image recognition, and others. This hardware can be easily implemented to metal cutting applications.

(b) The calculation speed of WT–NN would be 10–1000 times faster than any available procedure [1–8, 10, 11]. While all of the present techniques were developed for single processor computers by using compiled programs, the WT–NN would use parallel processors and assembly language programming for calculations.

(c) Time-series methods continuously collect data and update their models [6, 8]. The proposed technique may inspect the tool condition from the data of any 360 points at any desired time. Only the neural network-based techniques, or specially developed techniques, can satisfy this condition [1–3, 10, 11].

(d) The proposed method may be trained to evaluate the cutting process during the machining of an entire workpiece. Such a system will recognize initiation of cutting (transition), and all the other possible cutting conditions. However, in that case, the neural network will create thousands of cases and classifying each case by comparing it with so many patterns will take a long time. Most of the previous tool condition monitoring systems discuss indications of tool failure by using different approaches; however, they do not offer any automatic classification system for the output of their system [1, 5, 6]. All of the available methods would have similar problems (slow processing speeds) if they are used to monitor tool failure in all cutting conditions. After considering the above facts, it is recommended that the tool condition be evaluated at selected points of the machining operation. After the WT–NN system is trained on the data taken during the machining of the first workpiece at selected points, the tool condition can be evaluated from all subsequent workpieces at the same locations.

(e) The pattern of the resultant force is different each time the tool is broken. The ART2 neural networks create a new category when they encounter a totally new case. A well trained system can create a new category only when the tool is broken with a unique pattern. At the next tool failure, the system may indicate the problem if the pattern is the same; otherwise, the system will alert the operator to identify the recently created new category.

(f) WT–NN is a compression and classification technique. It may compress the information in any cutting condition without considering the cutting width, depth, orientation, and cutting parameters. Even for very complicated cases, such as when the tool removes material according to a very complex geometry (depth of cut is different since the tool is not perpendicular to the surface of workpiece, or several layers of a composite material are being cut at the same time), the proposed system is expected to identify the tool condition properly, if it was previously trained for the same conditions.

6. CONCLUSION

Use of wavelet transformations is proposed for the compression of resultant cutting forces in end milling operations. The estimated parameters of the wavelet transformations were classified with ART2-type neural networks.

The resultant force data of milling and drilling operations have a typical pattern that cannot be easily represented with harmonic functions. Wavelet transformations compressed the original data eight times and represented the important features of the signals after unnecessary details were eliminated. The variation of the estimated parameters of the wavelet transformations is very distinctive at different cutting conditions, and when the tool is broken.

The study indicates that neural networks can classify the estimated parameters of wavelet transformations accurately. At the selected vigilance (0.995), WT–NN recognizes the difference when a parameter of the cutting (feed, cutting speed and depth of cut) changes 50% or more, although the basic patterns remain the same. The system is capable of recognizing tool failure immediately, since ART2-type neural networks create a new category when they encounter a case they have not previously seen. The speed of the proposed system is expected to be 10–1000 times faster than any previously proposed system, if parallel processors and assembly language are used for the WT–NN hardware. The proposed system can also be trained to classify any complex cutting configuration, as long as the same conditions are encountered for all the workpieces, even when composite materials are cut.

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